UPDATED EDITION

Stress Analysis *of* Fiber-Reinforced Composite Materials

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Contributions on Fibers, Matrices, Interfaces,
and Manufacturing by Scott White
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DEStech Publications, Inc.

Stress Analysis of Fiber-Reinforced Composite Materials

DEStech Publications, Inc.
439 North Duke Street
Lancaster, Pennsylvania 17602 U.S.A.

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

Main entry under title:
Stress Analysis of Fiber-Reinforced Composite Materials

A DEStech Publications book
Bibliography: p.
Includes Index: p. 687

ISBN13: 978-1-932078-86-2

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Preface to the Updated Edition

This updated edition of *Stress Analysis of Fiber Reinforced Composite Materials* contains the same material as the original, with important exceptions. Typographical errors, identified by readers and reviewers, have been corrected. Changes were made to a number of figures to increase their clarity. Equations were modified to increase consistency throughout the text, and wording in many of the Exercises was edited to clarify what is being asked.

The book is intended as an introductory text for upper level undergraduate engineering students or first year graduate students. However, the book has proven to be useful for practicing engineers who find it necessary to understand the behavior of composite materials. The book emphasizes the mechanics of a stress and deformation analysis of fiber-reinforced materials as opposed to, for example, a materials science viewpoint. Exercises, including computer oriented exercises, are included within most chapters.

One key feature of the book, and one that sets it apart from other books on the subject, remains the series of example problems that are discussed throughout the text, starting with rather simple problems in Ch.4 that are then expanded upon in subsequent chapters. This series of problems uses the same material properties throughout so the impact of the elastic and thermal expansion properties for a single layer of fiber-reinforced material, in this case graphite-epoxy, on the stress, strains, elastic properties, thermal expansion, and failure stresses of cross-ply and angle-ply symmetric and unsymmetric laminates can be evaluated. Furthermore, calculations for various steps of the stress and deformation analysis of a laminate discussed in later chapters require combining simpler calculations from previous chapters, so using the same material properties throughout allows the example problems to conveniently build upon each other as the chapters progress and illustrate how concepts are linked. To provide examples of the unique and sometimes complicated deformation properties of fiber-reinforced materials, examples are sometimes reworked using the properties of aluminum, and the stresses and deformations of aluminum compared with those of graphite-epoxy. Users of the text have commented on how much they like the number and style of the example problems.

A second key feature is the consideration of thermally-induced effects due to curing, both in the context of laminate deformation, but also in the context of material failure. While the stresses and strains due to applied loads are calculated in many example problems, the reader is reminded that composite materials are cured at an elevated temperature and then cooled to a service temperature. The thermally-induced stresses and strains can add to or subtract from those due to applied loads.

Another feature, and one unique to this book, is the emphasis on the difference between specifying the applied loads, i.e., force and moment results, which is often the case in practice, versus specifying strains and curvatures and determining the subsequent stresses and force and moment resultants. This is a fundamental issue through all of solid mechanics and example problems included specifically to illustrate the difference are included.

As the example problems throughout the text represent the types of calculations that are required by researchers and designers working with fiber-reinforced composite materials, computer programming assignments are also included as recommended exercises in each chapter. As the calculations require coordinate transformations, frequent referencing of the fiber angle in each layer, computing strains in each layer, stresses in each layer, and the thermal effects in each layer, and are susceptible to algebraic errors, particularly as the number of layers in the laminate increases, it is recommended to the reader that key steps be programmed. Thus the simpler calculations of the early chapters can be programmed and checked relative to the example problems in those chapters, then those programming steps can be combined with other programming steps in later chapters and checked relative to the later more complicated example problems. The net result is a fairly comprehensive analysis tool authored by the reader and which can be changed as the need arises. This consideration of programming exercises, and checking them relative the example problems, has educational as well as practical advantages. For the interested reader, several programs written in FORTRAN are available which perform some of the calculations discussed in the book.

The chapter topics of this updated edition are the same as those outlined in the Preface to the Original Edition. Activities in the final chapter on manufacturing, e.g., hand lay-up, are designed to be carried out by students.

A note before closing. The Boeing 787 commercial transport and the very large wind turbines, such as those manufactured by Vestas Wind Systems, are but two examples of achieving designs virtually impossible without the use of fiber-reinforced composite materials. These examples have provided considerable motivation for updating this book, which was first published in 1998. Designs such as these are only possible by understanding the mechanical behavior of composite materials at the fundamental level, a primary purpose of this book and one which remains timely.

Finally, a note of thanks to DES*tech* Publications, Inc., materials science and engineering publishers, for the opportunity to continue to offer this book to the composites community, newcomers as well as establisher instructors and researchers. In particular, the encouragement and efforts of Anthony Deraco, DES*tech* president, Dr. Joseph L. Eckenrode, publisher, and Stephen Spangler, production manager, are much appreciated.

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September, 2008

Preface to Original Edition

Approach

This book focuses on the mechanics aspects of fiber-reinforced composite materials. By mechanics is meant the study of equilibrium, stress, strain, deformation, elastic properties, failure theories, and the linkages between these topics. A significant portion of the book emphasizes the use of mechanics to study the stresses due to applied deformations, loads, and temperature changes. Since interest in fiber-reinforced composite materials stems mainly from their ability to withstand high stress and deformation levels, such an emphasis centers on important issues.

No prior knowledge of composite materials is assumed. Only the basic concepts introduced in an undergraduate strength-of-materials course are necessary. The book is intended for use at the senior undergraduate or first-year graduate levels in any engineering curriculum designed to explore the behavior and performance of these advanced materials. Mechanical engineers interested in considering composite materials for automobiles or trucks and flywheels for energy storage, civil engineers investigating the application of composite materials to infrastructure, aerospace engineers studying advanced airframe design, and biomedical engineers developing lightweight composite materials for bone replacement and repair will find the book valuable.

A strong feature of the book is the use of a set of examples that is introduced early and then built upon as additional concepts are developed. This set of examples provides continuity to the discussion and allows the reader to evaluate the impact of more complicated issues on the stresses and deformations of fiber-reinforced composite materials as the book progresses. A second strong feature is the reminders of the implications of the various simplifying assumptions used to study the mechanical behavior of fiber-reinforced materials. These reminders are designed so the reader does not misinterpret the theories and results, and is able to evaluate outcomes based on the concepts presented. Many authors do not take the time or space to do this. A further helpful feature is the substantial coverage of thermal effects in composites, specifically the far-reaching effects of thermally induced deformations and stresses due to residual effects. With composite materials being envisioned for construction of high speed civilian airplanes, the coverage of thermal effects is timely. Also, a number of characteristics of fiber-reinforced materials are difficult to include in design and analysis procedures and are often difficult to fully comprehend. Specifically, the elastic couplings inherent in fiber-reinforced materials are often designed around, at some cost in design efficiency, or are assumed to be zero to simplify analysis procedures. This book counters this tendency by addressing the topic and by viewing elastic couplings as characteristics that can be used to aspire to designs not possible with metallic materials.

Because so many subtleties are involved with understanding and effectively using fiber reinforced composite materials, an in-depth view of a limited number of topics, rather than an overview of many topics, is offered. After working through this text, the reader will be well-versed in the details of important calculations and their impact on results. This book provides enough information so that students and engineers will know what questions to ask and find it easy to proceed to other resources on the subject.

Learning Aids

Many of the chapters include a list of suggested readings. Taken from well-known and readily available archival journals and books, the readings are selected to reinforce the principles presented in this book, expand on the concepts, and provide information on topics not discussed.

The notation used in this book is widely used. Important equations are enclosed in boxes for handy reference. One set of material properties, representing an off-the-shelf intermediate modulus polymer matrix graphite-fiber material, is used throughout. This feature, coupled with the continuing use of a set of examples, provides additional continuity from chapter to chapter. There are assigned exercise sets at the ends of many of the sections, which emphasize the fundamentals presented in that section. The exercises, though simple in the early chapters, become more involved as the book progresses. The exercises should be completed in conjunction with the sections where they are assigned, as opposed to reading ahead and then returning to previous sections to finish the exercises.

Computer exercises are also included. A number of steps for studying the response of composite materials are the same from one problem to the next. Programming these steps is recommended, and this activity forms the basis for these computer exercises. In this way it is possible to concentrate on the physics of the results, rather than on the algebra. In fact, the computer programming assignments are such that by the middle of the book the reader will have a computer program that can be used to predict some of the more important responses of composite materials, for example, stresses, strains, and thermal expansion coefficients. More importantly, because the programs are created by the reader, making changes, adapting them to special cases, changing the output, and so on, can easily be done, given the reader's knowledge of the unique situations to which the programs apply. In addition, the programs can be used to help complete some of the more complicated assigned exercises.

Contents

Chapter 1 provides a brief overview of the concept of fiber-reinforced materials—why fiber reinforcing can be used to achieve high-performance materials, and how the fiber and the material surrounding it, the matrix, interact. The chapter relies to a large degree on a materials science viewpoint to describe fibers, matrix materials, and fiber sizings. It is important for the mechanics minded specialist to be aware of the terminology and these basic ideas, particularly if working in an interdisciplinary environment.

Chapter 2 introduces the three-dimensional stress-strain behavior of a composite material that is used as the basis for discussion throughout the text. It is assumed that the fibers and matrix are smeared into a single homogeneous orthotropic material, and the chapter focuses on the response of a small, isolated element of this homogeneous material. The compliance and stiffness matrices are defined, and typical material properties for graphite reinforced and glass-reinforced materials are given. The chapter uses simple examples to emphasize the importance of a three-dimensional stress state. Also discussed is the response of an isolated element of material to temperature change.

Chapter 3 presents a brief overview of micromechanics. Unit cell models are studied with the aid of finite-element analyses. No finite-element theory is discussed; rather, emphasis is on the stresses within the unit cell as a function of fiber volume fraction. As a contrast to the numerically based finite-element models, the well-known concentric cylinders model, which is based on the theory of elasticity, is briefly introduced. Finally, several rule-of-mixture models are presented. One of the valuable results of Chapter 3 is that simple working expressions are developed to provide estimates of composite elastic and thermal expansion properties from the properties of the fiber and matrix.

As it is one of the most frequently used key assumptions in the analysis of the mechanical behavior of materials, the plane-stress assumption is the sole topic of Chapter 4. The three-dimensional stress-strain behavior of Chapter 2 is simplified to account for the plane-stress assumption, including thermal and moisture expansion effects. The consequences of these simplifications are emphasized with numerical examples.

Chapter 4 is coupled with, and leads directly into, Chapter 5, which discusses the plane stress stress-strain relations in a coordinate system not aligned with the principal material directions—a so-called global, or off-axis, coordinate system. Through simple examples, the response of an element of fiber-reinforced material with its fibers aligned at an angle relative to the coordinate system is described and quantified in detail. Counterpart examples using aluminum to dramatize the unusual response of composite materials are included. The engineering properties of an off-axis element of fiber-reinforced material are defined, and the coefficients of mutual influence are introduced.

Chapters 6, 7, and 8 constitute the central theme of the book, namely, the analysis of the response of composite laminates under the assumptions of classical lamination theory. Chapter 6 addresses another key assumption of the analysis of mechanical behavior of materials, the Kirchhoff hypothesis. Its implications are illustrated through a series of examples that are built upon in subsequent chapters. Because the Kirchhoff hypothesis is a kinematic assumption, its impact on the variation of the strains through the thickness of a laminate is first considered. Then, following the previously discussed plane-stress stress-strain relations, the stress variation through the thickness of the laminate is related to the strain variation through the thickness. The strains and stresses in the example problems are considered in detail. From the way the stresses are observed to vary through the thickness of the laminate, the definition of force and moment results seems natural. These quantities are informally defined, and the force and moment resultants for the example problems are computed strictly on physical grounds.

In Chapter 7, the force and moment resultants are formally defined, and as a result, the classical A, B, and D matrices are introduced. The calculation of the elements in each of these three matrices is detailed, and simplifications of the matrices for special but important laminates are presented. Considerable discussion is provided to interpret the physical meaning of the various off-diagonal terms in the A and D matrices, and the meaning of the B matrix.

Chapter 8 presents other examples of laminate response based on the assumptions of classical lamination theory. Force-based counterpart examples to the kinematics-based examples of Chapter 6 are presented, and the results contrasted. The emphasis on basic principles, such as differentiating between specifying kinematics and specifying forces or moments, is one of the powerful features of this book.

Chapters 9 and 10 introduce the topic of failure of fiber-reinforced composite materials. Chapter 9 introduces the maximum stress failure criterion. The various failure modes associated with fiber-reinforced composite materials are introduced. In addition, to illustrate the criterion, and as realistic examples of results, several cases involving both simple and combined loading of laminates are presented. Failure loads and failure modes are predicted.

Chapter 10 introduces the Tsai-Wu failure criterion. This criterion was chosen because it considers interaction of the stress components as a possible cause of failure, in contrast to the maximum stress criterion, which assumes failure is due to only one stress component. The Tsai-Wu criterion is used to predict the failure loads and modes for the same cases considered using the maximum stress criterion in the previous chapter. The results of the two criteria are contrasted.

Chapter 11 discusses the effect of a temperature change on the response of laminates. This is a major chapter in that it revisits classical lamination theory and the failure theories, but with the inclusion of thermal effects. Several of the example failure problems solved earlier without thermal effects are re-solved with thermal effects included. The thermal effect considered is the cooling from the consolidation temperature of the laminate, which is a residual thermal effect due to curing of the composite material.

Chapter 12 considers a topic that is often forgotten in books dealing with classical lamination theory—through-thickness strain effects. Through-thickness Poisson's ratios and coefficients of thermal expansion in the thickness direction are defined and illustrated in this brief and important chapter.

In Chapter 13 the mechanics of composite plates are introduced. The plate is assumed to obey all the assumptions of classical lamination theory, and the differential equations governing the plate are derived from equilibrium considerations. The boundary conditions that must be enforced along the edges are presented. For demonstrating some of the important effects found with composite plates, several semi-infinite plate problems that can be solved in closed form are considered. The influence of various boundary conditions and the coupling of boundary conditions with the B matrix for unsymmetric laminates are discussed. Finally, a finite, square, uniformly loaded, laminated plate is studied by using a series solution. With both the semi-infinite and square plates, stresses are also discussed. The point of the chapter is not a comprehensive view of composite plates; rather, the chapter serves as a bridge between the study of the response of the small, isolated elements of composite materials in the preceding chapters and the study of structural elements—the plate is one of the simplest.

Chapter 14, the Appendix, provides an overview of the manufacturing of composites. The fabrication and processing phases are considered, and a pictorial essay of the hand lay-up technique is provided so the important steps can be emphasized. The roles of release agents, peel plies, breather plies, and other specialty materials are described. The processing phase, with emphasis on autoclave processing, is briefly considered. Other forms of manufacturing, such as filament winding and pultrusion, are identified.

Supplements

An answer book for all the exercises is available for the instructor. Separate FORTRAN programs are available to compute the following: the engineering properties E_1, ν_{12}, E_2, and G_{12} from fiber and matrix properties; the components of the 6 x 6 compliance and stiffness matrices and the components of the 3 x 3 transformed reduced compliance and stiffness matrices, for the various stress-strain relations, from the engineering properties; laminate stiffness and thermal expansion properties from layer engineering properties and fiber orientations; and the stresses and strains through the thickness of a laminate due to given deformations, given force and moment resultants, or a given temperature change.

Acknowledgments

Before closing, I must mention several individuals and organizations that were of enormous help in developing this book. Scott R. White, of the Department of Aeronautical and Astronautical Engineering at the University of Illinois at Urbana Champaign, contributed the material in Chapter 1 on fibers, matrices, and interfaces, and the material in Chapter 14 on manufacturing. I am indebted to him for providing text and figures for these portions of the book. The contributions are vital to the book. At Virginia Tech Tamara Knott was an invaluable assistant for many of the calculations in Chapter 3. Caroline Scruggs also helped by assembling the calculations in graphical form. Beverly Williams provided considerable assistance with the text. And finally, the savior of many days and near disasters: Paula Davis. Paula worked the formatting and editing of this book into her very busy schedule. Without her this book probably would not exist. In addition, the very stimulating, positive, and always competitive atmosphere of the Department of Engineering Science and Mechanics at Virginia Tech had much to do with the composition of this book. Similarly, the research conducted within the NASA- Virginia Tech Composites Program has provided a foundation for this book. The students in the program, fellow faculty members in the program, and colleagues at the NASA–Langley Research Center all provided important stimuli.

Of immense help in refining the book were the reviewers who provided thousands of helpful comments. Their time and effort are greatly appreciated. The reviewers were: Donald F. Adams, University of Wyoming; Erian A. Armanios, Georgia Institute of Technology; Leif A. Carlsson, Florida Atlantic University; Charles E. Bakis, Pennsylvania State University; Sherrill B. Biggers, Jr., Clemson University; Paul F. Eastman, Brigham Young University; Harold E. Gascoigne, California Polytechnic State University at San Luis Obispo; Yousef Haik, Florida A&M University–Florida State University; Dahsin Liu, Michigan State University; Ozden O. Ochoa, Texas A&M University; Nicholas J. Salamon, Pennsylvania State University; Richard A. Scott, University of Michigan; David L. Sikarskie, Michigan Technological University; C. T. Sun, Purdue University; and Anthony J. Vizzini, University of Maryland.

Finally, the time, patience, and very helpful comments of the two editors involved in this text, John Corrigan and Debra Riegert, are a significant contribution. Editors must balance technical, editorial, production, and marketing issues. Their work is not easy and it is very much appreciated.

M. W. Hyer

CHAPTER 1

Fiber-Reinforced Composite Materials

1.1 Background and Brief Overview

Studies of strong, stiff, lightweight materials for application to diverse structures—from aircraft, spacecraft, submarines, and surface ships to robot components, prosthetic devices, civil structures, automobiles, trucks, and rail vehicles—focus on using fiber-reinforced materials. But why are fibers getting so much attention? To answer this question, one must know something about material science and, in particular, about the molecular bonds that hold matter together. Even though this book is devoted to the *mechanics* of composite materials, because the fiber form is such a central concern, we will begin this chapter with a presentation of the basic concepts in material science associated with fiber reinforcement. Figure 1.1 illustrates a basic unit of material. At the corners of the unit are atoms or molecules held in place by interatomic bonds. The figure shows that this basic unit of material has directionally dependent properties. To varying degrees, many common materials, including iron, copper, nickel, carbon, and boron, have directionally dependent properties, with the directional dependence being due to the strengths of the interatomic and intermolecular bonds. The bonds are stronger in some directions than in others, and the material unit is very stiff and exhibits considerable strength in the direction of the stronger bond. Unfortunately, the favorable properties found in one direction usually come at the expense of the properties in the other directions. In directions perpendicular to the stiff and strong direction, the material is much softer and weaker. Other properties like electrical conductivity and heat conduction can also be directionally dependent.

When material is processed and fabricated in bulk form (e.g., in manufacturing steel billets, you start with molten steel and pour it into a billet form), the units of material are more or less randomly oriented within the volume of material (see Figure 1.2). As a result of random orientation, the bulk

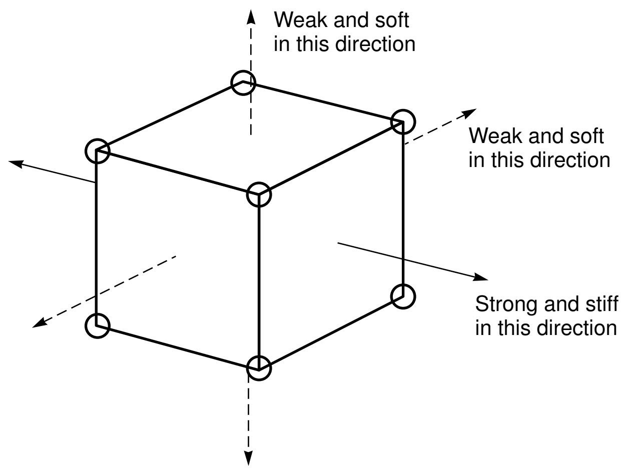


FIGURE 1.1. *Basic unit of material.*

material has the same properties in all directions. Generally, the properties of the bulk material reflect the poorer properties of the unit in Figure 1.1; the properties of the bulk material are determined more or less by the properties of the weakest link of the unit. Having the same properties in all directions is referred to as having isotropic behavior (the prefix *iso* means equal). Thus, the tensile strength of a specimen cut from a larger piece of steel or aluminum is independent of the direction the tensile specimen is machined from. (In a strict sense, this is not true; for instance, rolling metal into a sheet alters the

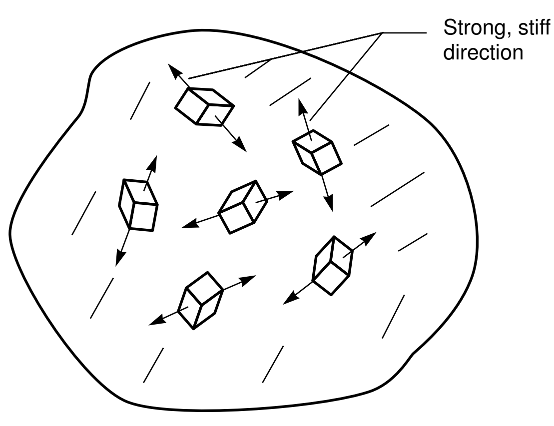


FIGURE 1.2. *Several basic units of material oriented randomly in bulk volume of material.*

microstructure of the material and causes it to have different properties in the roll direction than in directions perpendicular to the roll.)

If you can process the material in a manner that permits you to align the strong and stiff directions of all the basic material units, you can preserve some of the high strength and stiffness properties of a single unit, thereby countering isotropic behavior. Processing so that the strong and stiff directions of all the units align results in a long, thin element of material referred to as a whisker (see Figure 1.3). In reality, whiskers are quite small and, compared to bulk material, have very high strength and stiffness in the lengthwise direction. A typical whisker may be $1 - 10 \times 10^{-6}$ m in diameter and 10–100 times as long. With care in processing, the properties of microscopic whiskers can be very close to the ideal properties of a single unit. Unfortunately, attempting to lengthen a whisker by adding more basic units can cause imperfections and impurities. For crystal-like whiskers such as graphite, the imperfection may be a dislocation or the absence of a carbon atom. For polymeric fibers such as Kevlar®, foreign matter is a possibility. These deviations from ideal form significantly influence the strength and stiffness of the whisker and become the weak link in the material. Imperfections also cause an increase in both thermal and electrical resistance, leading to degraded conductive properties. Because there is no way to align them, whiskers used for reinforcement generally have a random orientation within the material, and so the reinforced material still has isotropic properties.

As more basic material units are added to the length of a whisker, it becomes what is called a fiber. Fibers have significant length, so they can be easily aligned in one direction to provide selective reinforcement within another material. A fiber contains many units in its length, and thus it has a greater chance of having an imperfection. As a result, a fiber is weaker than a whisker. The strength properties of fibers are a random variable. Testing 10,000 fibers would result in 10,000 different strength values. Obviously, you can use such raw strength data to form a probability distribution of the strength. The average strength and the scatter (variance) of the strength become important quantities in describing the properties of a fiber. Because of the random nature of fiber strengths, many researchers employ probabilistic methods to study the strengths of composite materials.

Some fibers, especially most forms of graphite fibers, have such small diameters that they are more conveniently handled in groups. A group of

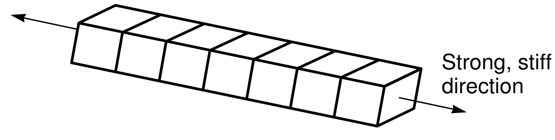
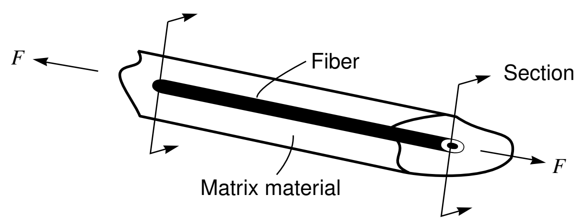


FIGURE 1.3. *Several basic units of material processed so their strong and stiff directions coincide.*

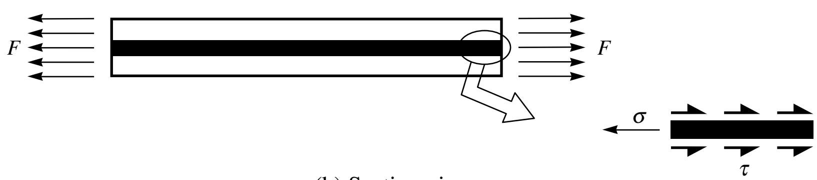
fibers is called a fiber tow, or simply a tow, and consists of from hundreds to hundreds of thousands of fibers. A tow is like a rope made of fibers, though generally without the complicated interlocking twist and braid patterns.

1.2 Utilizing the Strength of Fibers

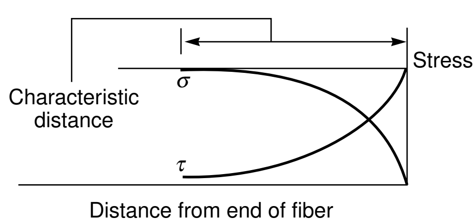
Once you can produce strong, stiff material in the form of fibers, there immediately comes the challenge of how to make use of the material: The fibers need to be aligned with the load, the load needs to be transferred into the fibers, and the fibers need to remain aligned under the load. Equally important, the fibers need to be in a format that makes them readily available and easy to use. Figure 1.4 illustrates the basic mechanism used to transfer a tensile load, F, into a fiber tow. Essentially, the fiber tow is embedded in, surrounded by, and bonded to another material; see Figure 1.4(a). The material, which is usually softer and weaker, not only surrounds the tow, but



(a) Fiber surrounded by matrix



(b) Section view



(c) Character of fiber stress distribution

FIGURE 1.4. *Load transfer to fiber: Tension.*

it also penetrates the tow and surrounds every fiber in the tow. The embedding material is referred to as the matrix material, or matrix. The matrix transmits the load to the fiber through a shear stress, τ. This can be seen in the section view, Figure 1.4(b), along the length of the fiber. Due to F, a shear stress acts on the outer surface of the fiber. This stress, in turn, causes a tensile stress, σ, within the fiber. Near the ends of the fiber the shear stress on the surface of the fiber is high and the tensile stress within the fiber is low. As indicated in Figure 1.4(c), as the distance from the end of the fiber increases, the shear stress decreases in magnitude and the tensile stress increases. After some length, sometimes referred to as the characteristic distance, the shear stress becomes very small and the tensile stress reaches a maximum value. This tensile condition continues along the length of the fiber. Generally, this characteristic distance is many times smaller than the length of the fiber.

For loading the fiber in compression (see Figure 1.5), the issue of fiber buckling must be addressed. If the shear stress on the end of the fiber in Figure 1.4(c) is reversed, then the stress within the fiber becomes compressive and attains a maximum value at some distance from the end. This is exactly like the tension case except that the fiber responds quite differently to a compressive load; specifically, the fiber tends to buckle. The compressive resistance of some types of fibers is so poor that they will kink and fold, much like a string loaded in compression. Other fibers are quite stiff and act

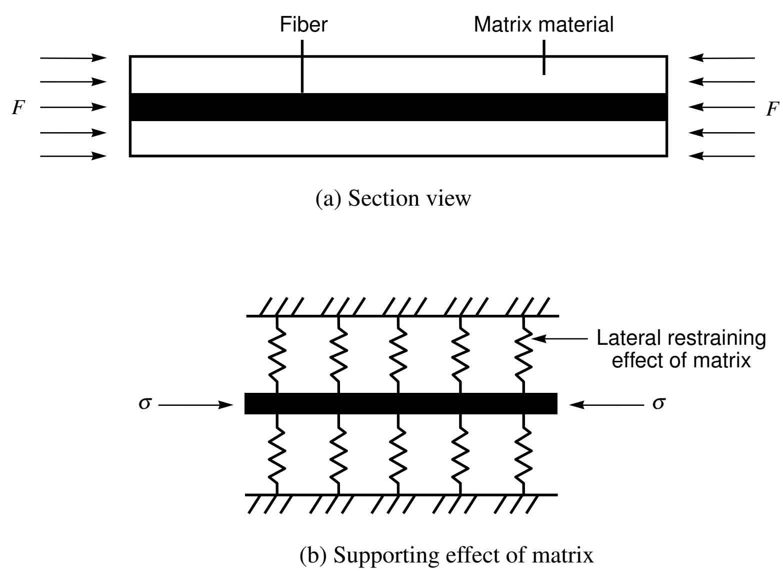


FIGURE 1.5. *Load transfer to fiber: Compression and the lateral restraint provided by the matrix.*

like very thin columns; they fail by what might be considered classic column buckling. To prevent the fiber from kinking, folding, or buckling due to a compressive load, it must be restrained laterally, and the matrix provides this restraint. To use a rough analogy, the fiber and matrix in compression are like a beam-column on an elastic foundation. As you might expect, in the presence of a compressive loading, any slight crookedness or waviness in the fiber can be quite detrimental.

Up to this point, we have focused on the idea of a single fiber or fiber tow, how loads are transmitted into it, and how it is prevented from buckling. The matrix serves both these roles. In addition, the matrix keeps the fibers aligned and in a parallel array. A cross section of a graphite-fiber-reinforced epoxy matrix composite is illustrated in Figure 1.6. The lighter circles are graphite fiber. Evident in Figure 1.6, in the upper left, is a region with no fibers, a so-called resin-rich region. Such regions can occur, and care should be taken to ensure they do not occur frequently. The embedding of strong, stiff fibers in a parallel array in a softer material results in a *fiber-reinforced composite material* with superior properties *in the fiber direction*. Clearly, the material properties perpendicular to the fiber direction are not as good. Recall that in an assemblage of basic units making up the fiber, as in Figure 1.3, the poorer properties of the basic unit are transverse to the lengthwise direction of the fiber. Therefore, to load a composite material perpendicularly to the fiber direction is to load the fiber in the soft and weak diametral direction of the fiber. In addition, if a composite material is loaded perpendicularly to the fiber direction, commonly referred to as the transverse direction, not all of the load is transmitted through the fiber. A portion of the load goes around the fiber and is entirely in the matrix material. This can be seen if it is imagined that the cross section of Figure 1.6 was subjected to horizontal tensile forces on the left and right edges of the figure. The fact that the fibers do not touch means some of the load must be transferred through the matrix. The poorer transverse properties of the fiber, coupled with the softer and weaker properties of the matrix, lead to poor properties of the composite in the direction perpendicular to the fibers. In addition, and more importantly, the transverse properties of the composite depend to a large degree on the integrity of the interface bond between the fibers and matrix. If this bond is weak, the transverse properties of the composite material are poor, and a poor interface leads to poor transverse strength. Progressive failure of the interfaces leads to what can be interpreted as low stiffness in the transverse direction. A poor interface results in high resistance to thermal and electrical conduction. Considerable research is directed toward improving the bond at the interface between the fiber and matrix by treating the surface of the fiber before it is combined with the matrix material to form a composite. Thus, as Figure 1.7 summarizes, though the use of fibers leads to large gains in the properties in one direction, the properties in the two perpendicular directions are greatly reduced. In addition, the strength

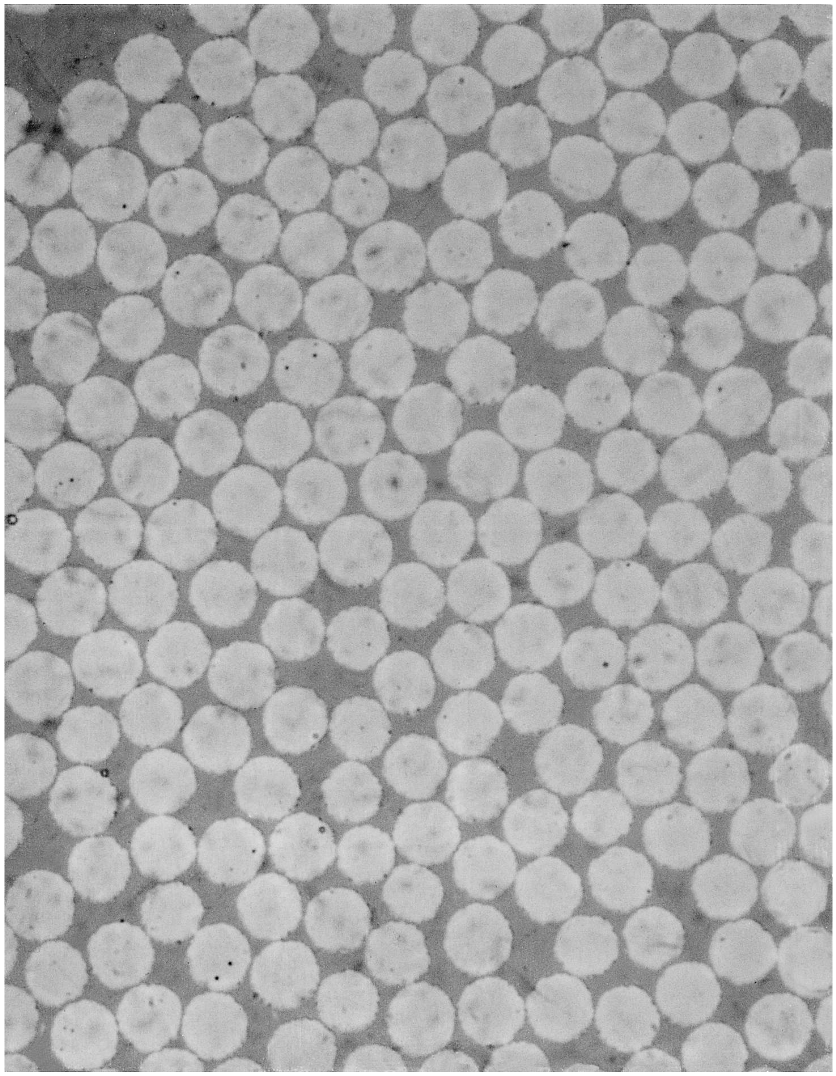


FIGURE 1.6. *Cross section of graphite-reinforced material.*

and stiffness properties of fiber-reinforced materials are poor in another important aspect. In Figure 1.8, the three basic components of shear stress are being applied to a small volume of fiber-reinforced material, but in neither case is the inherent strength of the fiber being utilized. In all three cases the strength of the composite depends critically upon the strength of the fiber-matrix interface, either in shear, as in Figure 1.8(a) and (c), or in tension, as in Figure 1.8(b). In addition, the strength of the matrix material

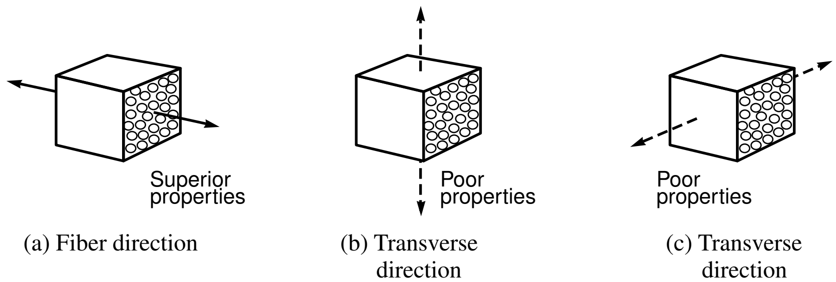


FIGURE 1.7. *Poor transverse properties.*

is being utilized to a large degree. This lack of good shear properties is as serious as the lack of good transverse properties. Because of their poor transverse and shear properties, and because of the way fiber-reinforced material is supplied, components made of fiber-reinforced composite are usually laminated by using a number of layers of fiber-reinforced material. The number of layers can vary from just a few to several hundred. In a single layer, sometimes referred to as a lamina, all the fibers are oriented in a specific direction. While the majority of the layers in a laminate have their fibers in the direction of the load, some layers have their fibers oriented specifically to counter the poor transverse and shear properties of fiber-reinforced materials. Despite these poor transverse properties, however, the *specific* strength, namely, strength normalized by density, and the *specific* stiffness, stiffness normalized by density, of composite materials are much greater than that of a single homogeneous material. Consequently, the weight of a structure utilizing fiber reinforcement to meet strength and stiffness requirements is reduced.

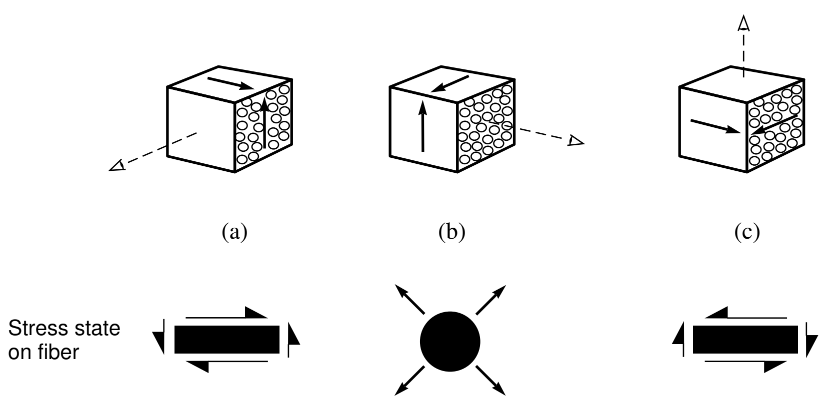


FIGURE 1.8. *Poor shear properties.*

1.3 Laminae and Laminates

Figure 1.9 shows a through-thickness cross section of an 11-layer flat laminate, with the layers not all having the same fiber orientation. This laminate, which is fabricated from carbon-based fibers in an epoxy matrix and is just over 3 mm thick, consists of five layers with their fibers oriented left and right on the page (the lighter strips), and six layers with their fibers oriented in and out of the page (the darker strips). For the layers with fibers perpendicular to the page, the tows have been sliced perpendicular to their length, while for the layers with fibers in the plane of the page, the tows have been sliced along their length. A closer view of the layers with their fibers oriented out of the page would look like Figure 1.6. This laminate is intended for use in a situation where there is slightly more load out of the page than there is across the page, and it would not be useful for shear loadings in the plane of the laminate. However, instead of having layers with fibers oriented at 90° to one another, if the fiber angles in some of the layers were oriented at 30°, 45°, or 60°, some inplane shear could be tolerated. A laminate subjected to both shear and tension may need fibers oriented at 45° to react the shear load, and at 0° and/or 90° to react the tensile load. The percentage of fibers to use in each orientation depends on the relative magnitudes of the tensile and shear loads.

The issues of fiber orientation, layers, and tension requirements versus shear requirements lead us to questions related to composite materials in a more general sense. How do we determine the fiber orientations for the best performance in a particular application? How many layers are required? How stiff should the fibers be? How strong? How detrimental are the poor transverse and shear strengths? To answer these questions we must develop the tools to help us understand the response of fiber-reinforced composite materials to applied loads. These tools will allow us to answer questions regarding the stress and strain states within a fiber-reinforced material. More importantly, they will allow us to identify the specific advantages of utilizing fiber-reinforced materials. However, these matters must be addressed in the

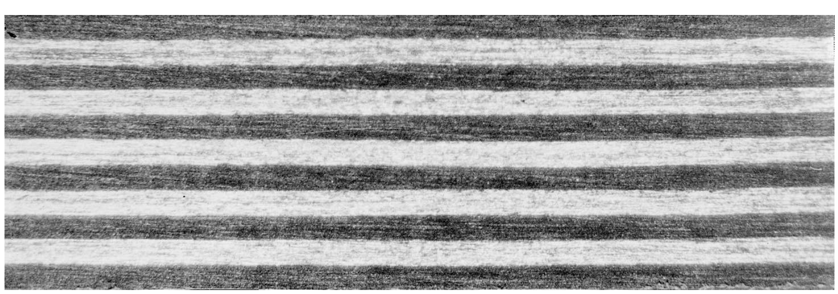


FIGURE 1.9. *Cross section of an 11-layer graphite-reinforced flat laminate.*

context of a particular application. For instance, one must consider the cost of using composite materials, including the cost of determining the final design. The cost to use composite materials may include education in the subject of composite materials, equipment to process and machine composite materials, and software to analyze composite materials. These are not trivial issues.

In the next chapter, we will begin laying the foundations that will allow us to answer many of these questions. Before proceeding, however, so that the foundations to be presented may be viewed in context, the next sections will present some of the important issues related to the production of fibers and the synthesis of matrix materials. Since most advanced composite materials are based on polymer matrices, we will emphasize these materials. Despite their high cost compared to polymer matrix materials, metals, such as aluminum or titanium, are sometimes used as a matrix, and many of the polymer matrix mechanics issues discussed in later chapters are equally valid for metal matrix composites.

1.4 Fibers

Boron fibers were used in early composite structures. Currently, three types of fiber reinforcements are in common use in polymer matrix composites, namely, carbon-based or graphite fibers, glass-based fibers, and synthetic polymeric fibers such as Kevlar. The basic building blocks for these three fibers are carbon, silicon, oxygen, and nitrogen, which are characterized by strong covalent interatomic bonds, low density, thermal stability, and relative abundance in nature.

1.4.1 Carbon-Based Fibers

To make carbon-based fibers, you begin with a precursor fiber. Early precursor fibers were made from commonly available rayon. Thornel 40®, from Union Carbide, and HMG-50® from Hitco are examples of early rayon-derived fibers. The yield of fiber from rayon precursor is relatively low. Currently, polyacrylonitril (PAN) precursor fibers are most commonly used. T300® from Toray and Type A® from the former Hercules are typical PAN-derived fibers. Precursor fibers made from pitch are also in use. Other precursors, such as phenolics, polyimides, and polyvinylalcohols, have been used but to a lesser degree. Ultimate fiber mechanical properties are not significantly affected by the type of precursor. However, the processing techniques are much different among the various precursors. In general, high-modulus carbon-based fibers are produced by carbonizing organic precursor fibers, and then graphitizing them at very high temperatures. Preferential orientation is achieved in the fibers by stretching them at various stages of processing. This stretching

Failure Theories for Fiber-Reinforced Materials: Maximum Stress Criterion

Failure of a structural component can be defined as the inability of the component to carry load. Though excessive deformation with the material still intact, as is the case for buckling, can certainly be considered failure in many situations, here failure will be considered to be the loss of integrity of the material itself. In the most basic sense, molecular bonds have been severed. If a fail-safe philosophy has been employed in the design of the structural component, then failure is not necessarily a catastrophic event. Rather, failure causes a load redistribution within the structure, a permanent deformation, or some other evidence that load levels have become excessive. The structure is still functional to a limited degree, but steps must be taken if continued use is to be considered.

Failure of fiber-reinforced materials is a complex and important topic, and studies of failure are an ongoing activity. For polymer-matrix composites, because the fiber direction is so strong relative to the other directions, it is clear that failure must be a function of the direction of the applied stress relative to the direction of the fibers. Causing failure of an element of material in the fiber direction requires significantly more stress than causing failure perpendicular to the fibers. Tensile failure in the fiber direction is controlled by fiber strength, while tensile failure perpendicular to the fibers is controlled by the strength of the bond between the fiber and matrix, and by the strength of the matrix itself. But what about the case of a tensile stress oriented at 30° relative to the fibers? We know that for this situation the stress component in the fiber direction, σ_1, the stress component perpendicular to the fibers, σ_2, and the shear stress, τ_{12}, can be determined using the stress transformation relations. Which stress component controls failure in this case? The stress component in the fiber direction? The stress component perpendicular to the fibers? The shear stress? Or is it a combination of all three? Because we are now in a position to calculate the stresses in the class of composite structures that satisfy the assumptions of classical lamination theory, it is appropriate to turn to the subject of failure and ask these questions.

There are many issues and controversies surrounding the subject of failure of composite materials. The matrix material of polymer matrix composites may be ductile and exhibit substantial yielding when subjected to high stress levels, and this yielding weakens support of the fiber, or degrades the mechanisms that transfer load into the fibers. On the other hand, the matrix material may be brittle and exhibit significant amounts of cracking around and between fibers as the stress level increases. This cracking will strongly influence the manner and efficiency with which load is transferred into the fibers, and strongly influence the performance of the material. In contrast, failure may be due to the fibers breaking or the fibers debonding and separating from the matrix. Subjected to a compressive load in the fiber direction, the fibers may buckle and deform excessively.

Clearly, we must consider many mechanisms when studying failure. In reality, failure is often a combination of several of these mechanisms, or modes. Failure can simply be the final event in a complex and difficult-to-understand process of damage initiation and accumulation within the material. A structure consists of multiple layers of fiber-reinforced materials, and even multiple materials, and there are multiple fiber directions and a range of load levels and load types. Consequently it is easy to understand why failure of fiber-reinforced composite structures is a difficult topic. Even with a single layer of material, the issues can be quite complicated. As a result, there have been many studies of failure. Each serious user of composite materials tends to develop their own philosophy about failure, based on the application, the material system, and their experience with testing and experimentation. Each large-scale commercial user of composite materials spends much time and capital gathering data to develop criteria and establish design stress levels.

While it is important to understand the mechanisms of failure, for many applications it is impossible to detail each step of the failure process. In the interest of utility, a failure criterion should be reducible to a level that can provide a means of judging whether or not a structure is safe from failure by knowing that a particular stress or combinations of stresses, or combination of strains, is less than some predetermined critical value. The failure criterion should be accurate without being overly conservative, it should be understandable by those using it, and it should be substantiated by experiment. A number of criteria have been proposed; some are rather straightforward and some are quite involved. The maximum stress criterion, maximum strain criterion, and failure criteria that account for interaction among the stress components are commonly used. This is because of the physical bases that underlie these criteria, particularly the maximum stress criterion and the maximum strain criterion. In addition, many of the criteria are simple variations of these, and the variations are based on experimental observation or on slightly different physical arguments put forth by the individuals identified with the criterion.

A legitimate question to ask at this point is, why are there a number of criteria? Isn't one sufficient? The answer is that no one criterion can accurately predict failure for all loading conditions and all composite materials. This is true for isotropic materials—some fail by yielding, others fail by brittle fracture. If we view failure criteria as indicators of failure rather than as predictors in an absolute sense, then having a number of criteria available, none of which covers all situations, becomes an acceptable situation.

In this book we will examine the maximum stress and the Tsai-Wu criteria. These two are chosen because they are among those commonly used for polymer-matrix composites. They represent a divergence of philosophies as to whether or not interaction between stress components is important in predicting failure. Also, by examining any particular criterion, we can present the issues that must be addressed when discussing failure of fiber-reinforced material. By incorporating a particular criterion into a stress analysis of a laminate, failure predictions are possible. The fact that several criteria are commonly used introduces the possibility of determining if using different criteria results in contradictory or similar predictions as to the stress levels permitted and the failure modes expected. Also, a detailed discussion of one or two basic criteria will allow you to form your own opinions regarding failure criteria.

As with the study of the stress-strain behavior of fiber-reinforced material, we shall approach the study of failure of a fiber-reinforced material by examining what happens to a small volume element of material when it is subjected to various components of stress. This is in keeping with the fact that stress is defined at a point, so logically we must assume that failure begins when certain conditions prevail at a point. We will continue to consider the fibers and matrix smeared into an equivalent homogeneous material when we are computing stresses, but to gain insight into the mechanisms that cause failure, it is useful to keep the separate constituents in mind. To that end, consider Figure 9.1; in the fiber direction, as a tensile load is applied, failure is due to fiber tensile fracture. One fiber breaks and the load is transferred through the matrix to the neighboring fibers. These fibers are overloaded, and with the small increase in load, they fail. As the load is increased, more fibers fail and more load is transferred to the unfailed fibers, which take a disproportionate share of the load. The surrounding matrix material certainly cannot sustain the load and so fibers begin to fail in succession; the failure propagates rapidly with increasing load. As with many fracture processes, the tensile strength of graphite fiber, for example, varies from fiber to fiber; and along the length of a fiber. The tensile strength of a fiber is a probabilistic quantity, and the mean value and its variance are important statistics. It is possible to study the failure of fiber-reinforced composites from this viewpoint, or to simply use a value of failure stress in the fiber

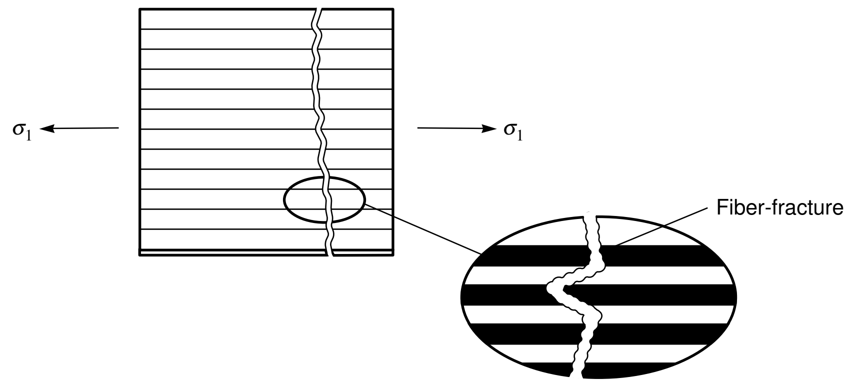


FIGURE 9.1. Failure in tension in the 1 direction.

direction that includes a high percentage of the fiber failure strengths. That will be the approach here. The tensile strength in the fiber direction will be denoted as σ_1^T.

With a compressive stress in the fiber direction, contemporary polymer-matrix composites fail by fiber kinking, or microbuckling, as in Figure 9.2. Kinking occurs among localized bands, or groups, of fibers, and the fibers in the band fracture at both ends of the kink; the fracture inclination angle is denoted as β, which varies from 10 to 30° in most composites. The width of the kink band, W, varies from 10 to 15 fiber diameters. The primary mechanism responsible for this behavior is yielding, or softening, of the matrix as the stresses within it increase to suppress fiber buckling. As might be suspected, any initial fiber waviness or misalignment, denoted as $\bar{\phi}$ in Figure 9.2, greatly enhances kinking and reduces fiber-direction compressive strength, σ_1^C. In well-made composites, $\bar{\phi}$ is typically between 1 and 4° (0.017–0.070 rads). The study of kinking is an ongoing topic of research, where such issues as the magnitude of the fiber modulus relative to the magnitude of the matrix modulus, bending effects in the fiber, the variance of the misalignment angle, and the influence of other stresses (say, a compressive σ_2), are being studied. It is generally accepted, however, that fiber misalignment and yielding of the matrix influence composite compressive strength in the fiber direction, σ_1^C, by way of the relation

$$\sigma_1^C = \frac{G_{12}}{1 + \dfrac{\bar{\phi}}{\gamma_{12}^Y}} \tag{9.1}$$

where γ_{12}^Y is the shear strain at which the composite shear stress–shear strain relation loses validity due to softening, or yielding, effects in the matrix. Values of $\bar{\phi}/\gamma_{12}^Y$ range from 2 to 6, depending on the material.

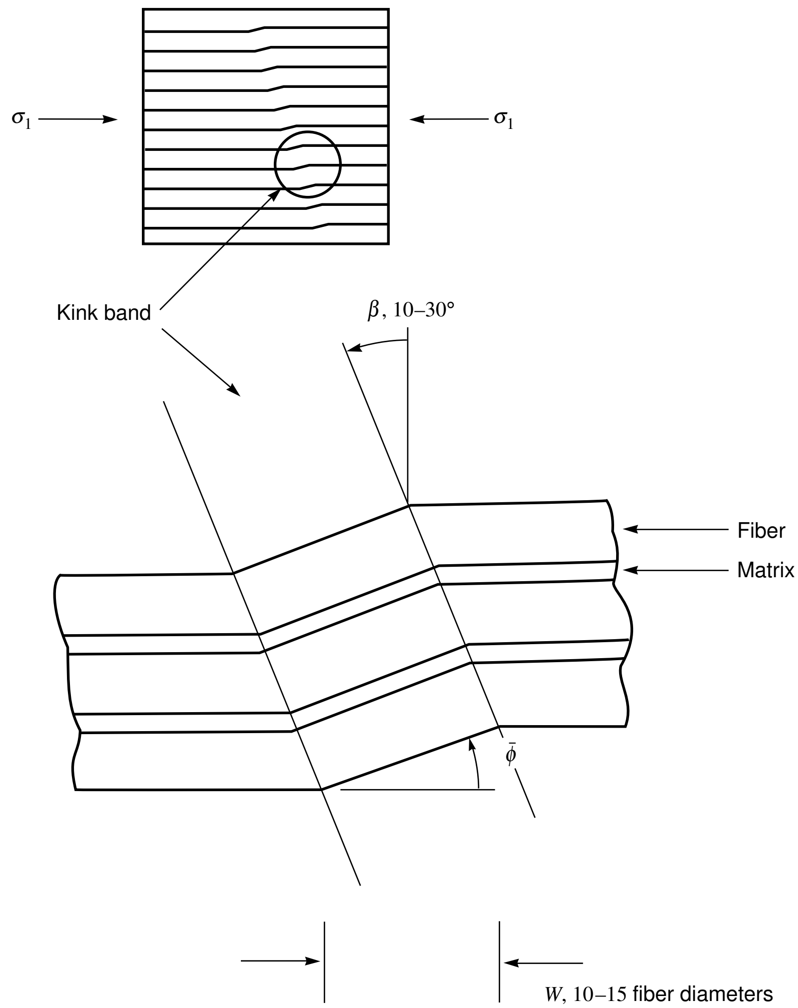


FIGURE 9.2. *Failure in compression in the 1 direction.*

Often polymeric fibers in a polymeric matrix fail in compression due to fiber crushing rather than fiber kinking. The compressive stresses in the fiber cause the fiber to fail before the matrix softens enough to allow kinking.

As illustrated in Figure 9.3, perpendicular to the fiber, say, in the 2 direction, failure could be due to a variety of mechanisms, depending on the exact matrix material and the exact fiber. Generally a tensile failure perpendicular to the fiber is due to a combination of three possible micromechanical failures: tensile failure of the matrix material; tensile failure of the fiber across its diameter, and failure of the interface between the fiber and matrix. The latter

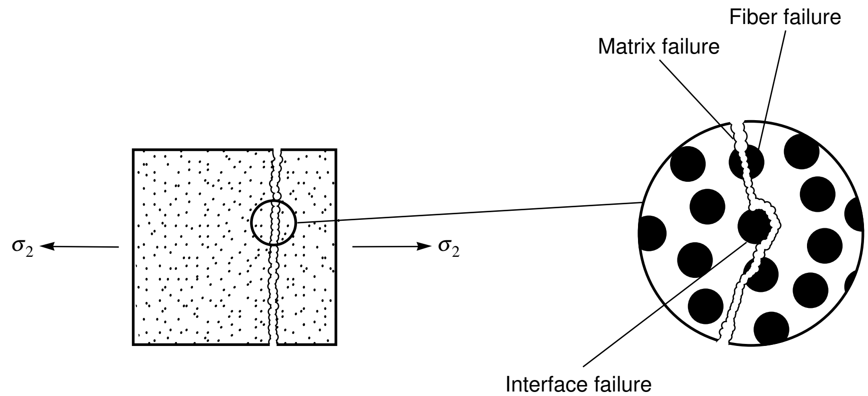


FIGURE 9.3. *Failure in tension in the 2 direction.*

failure is more serious and indicates that the fiber and matrix are not well bonded. However, due to the chemistry of bonding, it is not always possible to have complete control of this bond.

Failure in compression perpendicular to the fibers, as in Figure 9.4, is generally due to material crushing, the fibers and matrix crushing and interacting. The compressive failure stress perpendicular to the fibers is higher than the tensile failure stress in that direction. Herein the tensile failure stress perpendicular to the fibers will be denoted as σ_2^T, while the compressive failure stress will be denoted as σ_2^C. Failure in the 3 direction is similar to failure in the 2 direction and the failure stresses will be denoted as σ_3^T and σ_3^C.

The shear strength in the 2-3 plane, denoted as τ_{23}^F, is limited by the same mechanisms that govern tensile strengths perpendicular to the fibers, namely, matrix tensile failure, failure across the diameter of the fiber, and interfacial strength. Because a shear stress produces a tensile stress on a plane oriented at 45°, these tensile micromechanisms again limit the performance of the material. Figure 9.5 illustrates these mechanisms as viewed in this shear

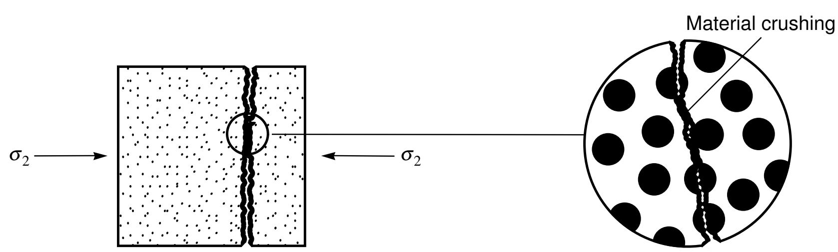


FIGURE 9.4. *Failure in compression in the 2 direction.*

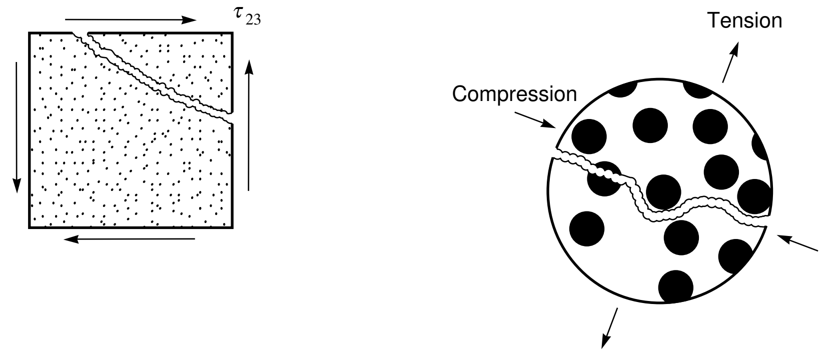


FIGURE 9.5. *Failure in shear in the 2-3 plane.*

mode. Because of these mechanisms that control shear strength, the shear strength in the 2-3 plane is independent of the sign of the shear stress.

The shear strength in the 1-2 plane is limited by the shear strength of the matrix, the shear strength of the fiber, and the interfacial shear strength between the fiber and matrix. Figure 9.6 depicts failure in shear in the 1-2 plane due to a shear separation of the fiber from the matrix along the length of the interface. Failure in the 1-3 plane follows similar reasoning, and these shear strengths are denoted as τ_{12}^F and τ_{13}^F. As expected, the failure strength in this plane is independent of the sign of the shear stress.

In summary, then, for a fiber-reinforced composite material there are nine fundamental failure stresses to be concerned with, six normal stresses, three

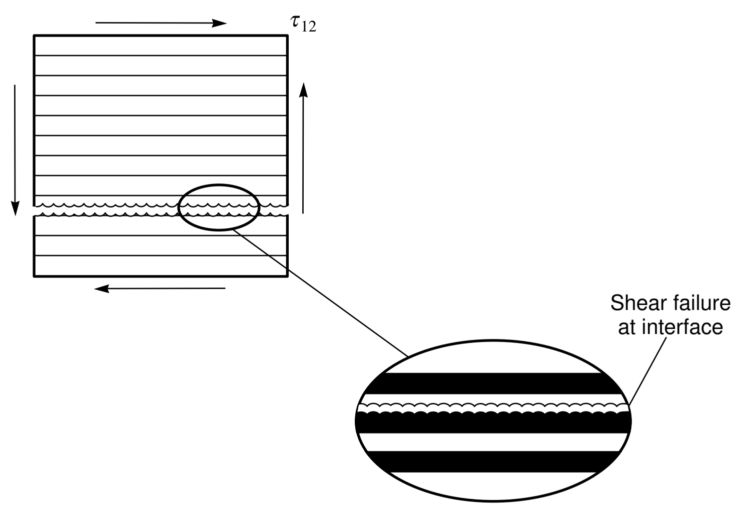


FIGURE 9.6. *Failure in shear in the 1-2 plane.*

tensile and three compression, and three shear stresses. Each failure stress represents distinct microfailure mechanisms, and each results in a failure that is somewhat unique to that loading situation. Measuring these failure stresses is difficult. Many issues are involved with the testing of fiber-reinforced composites to determine failure stresses; most of them focus on the fixturing and specimen shape and dimensions. There is also the issue of interaction among stress components. We have indicated that compression failure in the fiber direction is due to kinking and microbuckling, and it stands to reason that a compressive stress perpendicular to the fibers—that is, a σ_2 or σ_3—might help support the fibers and prevent, or at least delay, microbuckling and increase the compressive load capacity in the fiber direction. However, testing to study interaction between stress components is difficult. To determine if a compressive stress perpendicular to the fibers increases the compressive strength in the fiber direction, a fixture to vary the level of compressive σ_2 and then the level of compressive σ_1 would have to be constructed and a range of load levels used. The construction of such a biaxial compressive test fixture is difficult. And what of triaxial compression? If a compressive σ_2 might increase the compressive capacity in the fiber direction, what about a combination of compressive σ_3 and a compressive σ_2? And what about the possible interaction of tension perpendicular to the fibers and shear? From Figures 9.3 and 9.5 we saw that the same mechanisms that are responsible for limiting the value of τ_{23} limit the value of a tensile σ_2, so it is conceivable that these two stress components interact to influence failure. The possibilities are enormous. Unfortunately, it is quite easy to generate failure criteria that are too complex to be verified experimentally. An important requirement of any failure criterion is to be able to conduct failure tests on simple specimens subjected to fundamental stress states, and then be able to predict the load levels required to produce failure in more complicated structures with more complex stress states. The two criteria considered here rely on the fundamental failure strengths discussed above. The maximum stress criterion is a noninteractive failure theory, while the Tsai-Wu criterion is an interactive theory, and the influence of stress component interaction will be observed.

Because the majority of what we have discussed so far has been devoted to situations where the plane-stress assumption has been used, we shall limit our discussion of failure to those cases also. Hence we shall be interested in the following five failure stresses:

$$
\begin{aligned}
\sigma_1^C &: \text{ compression failure stress in the 1 direction} \\
\sigma_1^T &: \text{ tensile failure stress in the 1 direction} \\
\sigma_2^C &: \text{ compressive failure stresses in the 2 direction} \\
\sigma_2^T &: \text{ tensile failure stress in the 2 direction} \\
\tau_{12}^F &: \text{ shear failure stress in the 1-2 plane}
\end{aligned}
\tag{9.2}
$$

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