

# THE MECHANICS OF ADHESIVES IN COMPOSITE AND METAL JOINTS

*Finite Element Analysis with ANSYS*

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**DEStech Publications, Inc.**

## **The Mechanics of Adhesives in Composite and Metal Joints**

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439 North Duke Street  
Lancaster, Pennsylvania 17602 U.S.A.

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Printed in the United States of America  
10 9 8 7 6 5 4 3 2 1

Main entry under title:

The Mechanics of Adhesives in Composite and Metal Joints: Finite Element Analysis with ANSYS

A DEStech Publications book  
Bibliography: p.  
Includes index p. 215

Library of Congress Control Number: 2014936350  
ISBN No. 978-1-60595-096-9

# Preface

**A**DHESIVE BONDING TECHNOLOGY is a powerful joining technique, especially for thin sheets of metal or composites. The superiority of adhesive bonding is manifest in its high fatigue resistance and high strength-to-weight ratio. Since an adhesively bonded joint consists of different materials, its structural analysis is complicated and requires many special considerations and assumptions. For instance, when adhesives are used to join thin sheets, large deformation behavior is expected under thermal and mechanical loads. In addition, modern adhesives display significant degrees of plasticity, which further complicates analysis. For an analysis of this kind that includes diverse materials and geometric non-linearities, a reliable analytical solution is almost impossible.

Numerical techniques, such as Finite Element Analysis (FEA), offer an efficient and powerful solution for analyzing complicated structures under varying loading conditions, such as those in adhesively bonded joints. FEA can also be used for other types of analyses, e.g., stress, thermal, and diffusion, which are often required to study the behavior and responses of bonded joints during their service life. In the last few decades, rapid advances in FEA technology have led to the development of commercial FEA packages. One of the packages most widely used by engineers is ANSYS.

This book concentrates on studying the mechanics of adhesively bonded composite and metallic joints using FEA, and more specifically, the ANSYS package. The main objective of the book is to provide engineers and scientists working in adhesive bonding technology with the technical know-how to model adhesively bonded joints using ANSYS.

The text can also be used for post-graduate courses in adhesive bonding technology. It also provides fundamental scientific information regarding the theory required to understand FEA simulations and results. The types of problems considered herein are: stress, fracture, cohesive zone modeling (CZM), fatigue crack propagation, thermal, diffusion and coupled field analysis.

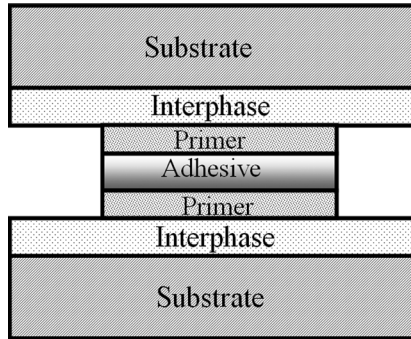
Chapter 1 presents a brief history of adhesive bonding, as well as its applications and classifications. The second chapter is devoted to reviewing basic mechanics theories used in the following chapters, including stress and strain, plasticity, fracture mechanics, heat transfer, and diffusion. Chapter 3 covers the fundamentals of FEA and introduces the ANSYS package. The theoretical background of structural mechanics, heat transfer and diffusion problems is explained. Element types, as well as FEA formulations, are considered. Chapter 4 concentrates on defining element types, material models and constructing the FE mesh for several types of un-cracked and cracked adhesive joints. Modeling damage in bonded joints using CZM is also considered, and the models developed in Chapter 4 are then used to perform different types of analyses in Chapters 5 through 9. In Chapter 5, stress analysis for four different joints is presented, while fracture and CZM analyses are explained in Chapter 6. The seventh chapter focuses on fatigue crack propagation analysis and lifetime prediction of two adhesively bonded joints. Thermal and diffusion analyses of three different joints are explained in Chapter 8. Finally, in Chapter 9, coupled thermal-stress and diffusion-stress analyses are carried out. All ANSYS input files described in the chapters of the book are also available in electronic files provided with the book.

# An Introduction to Adhesive Joints

## 1.1. INTRODUCTION

**A**DHESIVE is defined as a substance that is capable of strongly and permanently holding two surfaces together. Bonding is the joining of the two materials, known as substrates or adherends, using an adhesive material. The terms substrate and adherend are synonymously used in the literature, although sometimes the term substrate refers to the material before bonding and the term adherend after bonding. For convenience and to avoid confusion, we shall use the term substrate throughout the book. The adhesive material adheres to the substrates and transfers the forces between them. In general, the bonding will not be broken unless the bond is destroyed. An example of a typical adhesively bonded joint is shown in Figure 1.1, from which different regions can be identified. The interphase is a thin region near the contact between adhesive and substrate and has different physical and chemical properties from adhesive and substrate materials. The term interphase is to be distinguished from the term interface, which is the plane of contact between the surfaces of two materials. A second region that can be seen in Figure 1.1 is the primer, which is applied to the surface prior to the application of an adhesive. Although not always used, a primer improves the performance of bond and protects the surface until the adhesive is applied.

Nowadays, adhesive bonding becomes the most universal joining technique as it can be used to join any type of materials. Consequently, adhesive bonding joining technique gains lots of popularity because it offers flexible design and can have a wide range of industrial applica-



*FIGURE 1.1. A typical adhesively bonded joint.*

tions. It is replacing traditional joining techniques in many applications. With the advances of polymer chemistry, modern adhesives may have high strength and short curing time. Therefore, a very strong adhesively bonded joint can be obtained in a very short time. Adhesive is suitable for joining thin sheets and this is the reason why it becomes very popular in aerospace and automotive industries, where light weight is of primary importance. Adhesive bonding has many advantages, which are summarized as follows:

1. It offers the possibility to join large surfaces, dissimilar materials and thin substrates.
2. It provides good uniform load distribution, except at edges.
3. It does not make any visible surface marking.
4. It has excellent fatigue performance.
5. It has good damping and vibration properties.
6. It requires low heat so that substrates are not affected.
7. It provides high strength to weight ratio.

However, adhesive bonding has several disadvantages, which are summarized as follows:

1. Cleaning and surface pre-treatment is required in order to achieve high quality bonding.
2. Long curing periods may be required.
3. Pressure and fixtures may be required.
4. Inspection of joints after bonding is difficult.
5. It is sensitive to high temperature and moisture concentration.
6. Special training may be required.

A comparison between three different joining techniques, namely riveting, welding and adhesive bonding, is presented in Table 1.1, which summarizes the advantages and disadvantages of each technique and can be used to identify which fastening method is suitable for a particular application. For example, for thin metal structures used in aerospace and automotive industries, the transmission of stresses is more effective by adhesive bonding than by riveting or welding joining methods. Materials such as plastics, Fibre Reinforced Polymer (FRP) composites and elastomers are easier joined by adhesives than by other techniques. Welding is sometimes difficult for light metals such as aluminium, titanium and magnesium due to the high level of heat, and therefore adhesive bonding provides a good alternative for joining them. One of the main advantages of adhesive bonding is its excellent fatigue performance when compared to other joining techniques [1]. In Figure 1.2, a sketch of stress versus number of cycles, S-N curve, for a metal substrate, a riveted joint and an adhesive joint is shown. The fatigue resistance of the adhesive joint is far better than that of the riveted joint and close to that of the metal substrate.

The modes of failure in an adhesively bonded joint can be cohesive, adhesive or a combination of cohesive/adhesive failures as shown in Figure 1.3. Cohesive failure, Figure 1.3(a) can be either in the adhesive layer or in the substrate. In the example given in Figure 1.3(a), cohesive failure is in the adhesive layer, which may take place when the interface is stronger than the adhesive material. Adhesive failure, Figure

**TABLE 1.1. Comparison Between Riveting, Welding and Adhesive Bonding Techniques.**

Characteristic	Riveting	Welding	Adhesive Bonding
Joining thin materials	Poor	Fair	Excellent
Limits on material combination	Fair	Poor	Excellent
Requirement for surface preparation	Excellent	Good	Poor
Tooling	Excellent	Fair	Fair
Heat requirement	Excellent	Poor	Good/Fair
Stress distribution	Poor	Good/Fair	Excellent
Sealing function	Poor	Fair	Good
Distortion assembly	Fair	Poor	Excellent
Solvent resistance	Excellent	Excellent	Fair
Effect of temperature	Excellent	Excellent	Poor
Ease of repair	Good	Poor	Fair
Level of required skills	Excellent	Good	Excellent

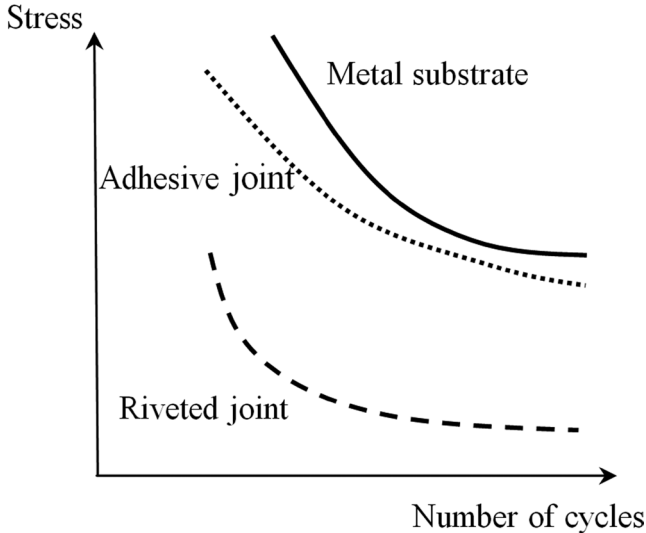


FIGURE 1.2. S-N curves for metal substrate, riveted joint and adhesive joint, adopted from [1].

1.3(b), also known as interfacial failure, takes place when the interface is weaker than the adhesive material and represents a failure of the bond between adhesive and substrate. A combination of cohesive/adhesive failure also is possible as shown in Figure 1.3(c).

Adhesive bonding technology is a multi-disciplinary science that requires the knowledge of a number of scientific disciplines as illustrated in Figure 1.4. Three main academic disciplines, namely mechanics, physics and chemistry are overlapping to produce important research topics, such as adhesion science, polymer science, surface science and

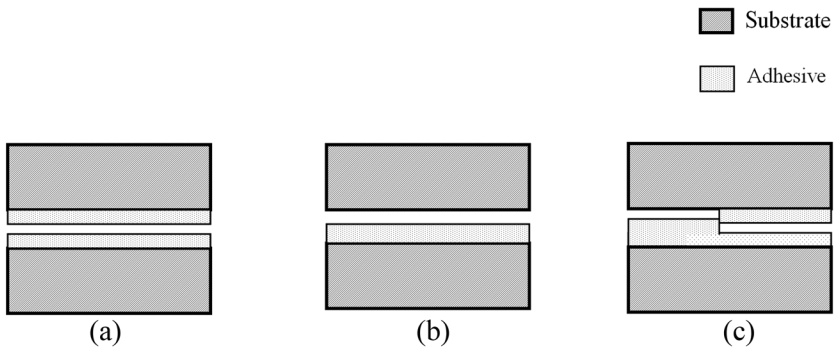
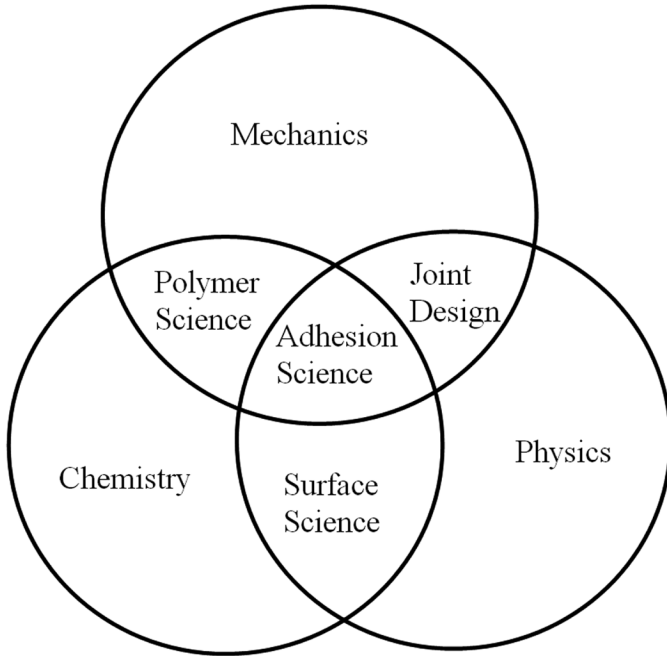


FIGURE 1.3. Typical failures in adhesive; (a) cohesive failure, (b) adhesive failure and (c) cohesive/adhesive failure.





*FIGURE 1.4. Multi-disciplinary aspects of adhesive bonding technology.*

joint design. This book concentrates on the mechanics aspect of adhesive joints and more specifically stress analysis, fracture and damage mechanics, thermal, diffusion and coupled analyses using Finite Element Analysis (FEA) technique.

## **1.2. A BRIEF HISTORY OF ADHESIVE BONDING**

As an Egyptian, I am proud to say that ancient Egyptians were among the early humans in the ancient ages who made use of adhesives. In the tomb of Rekhmara in Tibah, which dates to 1475 B.C., animal glues were used in a wall carving. In the tomb of Tut-an-khamun discovered in 1922 in the Valley of the Kings, a glue tablet was found. Surprisingly, the glue's properties were found to be identical to those at the time of the archaeological investigations indicating that adhesives have not been further developed since the time of ancient Egyptians. Egyptians used glues in many applications including fastening gold leaf to plaster, fastening wood, sealing and repairing alabaster jars, compound bow and as a binder in paints and pigments.

Although I have started with ancient Egyptians, the history of adhe-

sives is much older than that. It is very difficult indeed to trace the exact starting date for the use of adhesives. It might have been started at the same time as the existence of human being. Archaeological evidence suggests that humans have used adhesives for thousands of years, dating back approximately 200,000 B.C. In Koenigsau in the Harz Mountains in Germany in 1963, residues of adhesives were found on Neanderthal tools dated to approximately 80,000 B.C. Other Neanderthal tools dated to 40,000 B.C. have been found in Umm el Tiel in Syria. Adhesives used by modern humans have been dated to 8,000 B.C. Statues discovered in Babylonian temples contain glues and have been dated to 4,000 B.C. The Sumerians in 3000 B.C. used glue produced from animal skins and the Mesopotamians in 4000 B.C. used asphalt. In 1991, a discovery revealed adhesives were used to bond components of weapons from the Late Neolithic period dated in 3,300 B.C. During the period between 2000 B.C. and 1600 B.C., ancient Greeks used glues in the famous legend of Daedalus and Icarus. The first bonding of structural metal probably was done by the ancient Greek sculptor and architect Theodorus of Samos (from the Greek island of Samos) and is dated to 530 B.C.

In the middle ages, immediately after the decline of Greece and Rome empire, very few records documenting the use of adhesives can be found. It is very likely that adhesives were in use during several centuries. The use of adhesives restarted in the 16th century for inlaying work and further in the 17th century for veneering. In the 18th century, adhesives were used in the production of furniture. In 1690, the production and practical manufacturing of glues started in the Netherlands and moved to England in 1700. The first patent related to glue, titled “a kind of glue called fish glue”, was published in Britain in 1754, followed by other patents related to animal glues during the next few hundred years. During this period, animal and vegetable glues were used to bond wood and paper products. By the end of the 19th century and the beginning of the 20th century, many publications appeared to share knowledge of glue use, manufacturing and testing. Advances were noticeable in many issues including glue production on industrial scale, importance of quality control and testing of adhesive products. By around 1920, the use of adhesives in the manufacturing of aircraft and automobile has been started. The adhesives available at that time were of nature origin [2], namely animal glue, fish glue, liquid glue or animal glue in liquid, marine glue made from indiarubber, naphtha and shellac, casein glue, waterproof glue, vegetable glue, flexible glue (modified animal glue) and albumen glues.

Alexander Parkes introduced celluloid in 1862 and started the development of synthetic polymers, which had a significant effect on the history of adhesives. In 1872, Baeyer has used Phenol-formaldehydes to produce resins for the first time, followed in 1905 by Leo Baekeland, who introduced a commercial product of a synthetic resin called “Bakelite.” In 1930, a commercial product of phenolic resin that can be used in the manufacturing of polywood was made available. Later on, phenolic adhesives were developed in water emulsions and dry powders. The history of the development of Phenol-formaldehydes is summarized in Table 1.2 [3]. In 1918, Hans John proposed the use of Urea-formaldehyde as adhesives. The developments of polyvinyl acetate, polyvinyl chloride and acrylic adhesives took place around 1912 by synthesise and polymerise vinyl acetate and vinyl chloride monomers. Acrylic polymers formed the basis of anaerobics, ultraviolet hardening and two part toughened adhesives. In 1937, Otto Bayer published a patent on isocyanate polyaddition process and developed Polyurethane polymers. Polyurethane adhesives have been used for bonding glass, wood, composite, rubber and leather. In 1936, Pierre Castan has introduced epoxy resins, which can be considered as one of the most important product in the history of adhesives [4]. He produced the first synthesised epoxy resins. In 1939, Greenlee produced epoxy resins using epichlorhydrin and bisphenol A. In 1946, Swiss Industries Fair developed four electrical casting resins for commercial exploitation of epoxy adhesive. Due to their versatility, good mechanical properties and ease of use, epoxy adhesives are nowadays used in many industries including aerospace, automotive, electronics and construction. They have high shear strength, but low toughness and peel stress. In order to improve their properties, the use of additives has been proposed. In 1970, butadiene-based rubber modifiers from Goodrich was introduced to improve peel, impact and fatigue resistance.

**TABLE 1.2. Historical Development of Adhesives [3].**

<b>Year of Availability</b>	<b>Adhesive Type</b>
1910	Phenol-formaldehyde
1930	Urea-formaldehyde
1940	Nitrile-phenolic, vinyl-phenolic, acrylic, polyurethane
1950	Epoxies, cyanoacrylates, anaerobics
1960	Polyimide, polybenzimidazole, polyquinoxaline
1970	Second-generation acrylic

### 1.3. CLASSIFICATION OF ADHESIVES

Adhesives may be classified in many different ways. The classification of adhesives is quite important for the selection of a proper adhesive for a certain application. Today, a large number of adhesive types is available for engineers. This makes the selection of a proper adhesive quite a difficult task. The common classifications used in the industry are by: (1) function, (2) chemical composition, (3) method of reaction, (4) physical form, (5) cost and (6) end use. Table 1.3 summarized the different ways to classify adhesives.

#### 1.3.1. Classification by Function

Adhesives are classified by function as structural and non-structural. Structural adhesives are materials with high strength, which bond structures and resist loads during the service life and in the designed operating environments. Non-structural adhesives bond lightweight materials in place and are not subjected to high external loads. They are used for temporary short term fastening and as a secondary fastener in a hybrid (e.g. bonded/bolted) joint. Examples of non-structural adhesives include hot melt and water emulsion adhesives.

**TABLE 1.3. Classification of Adhesives.**

	<b>Classification</b>
Function	Structural Non-structural
Chemical composition	Thermosetting Thermoplastic Elastomeric Hybrid
Method of reaction	Chemical reaction Loss of solvent Loss of water Cooling from melting
Physical form	Solid 100% solid paste and liquid 100% solid paste and liquid with solvent to reduce viscosity
Cost	Including labor, equipments, curing time, loss due to defective joints
End use	Substrate type Environments

### 1.3.2. Classification by Chemical Composition

Adhesives are classified by chemical composition as thermosetting, thermoplastic, elastomeric or combination of them (hybrid). Thermosetting adhesives cannot be heated and softened after initial cure that takes place by an irreversible chemical reaction at room or elevated temperature. Examples of thermosetting adhesives are epoxy and urethane. Thermoplastic adhesives are materials that do not cure or heated. They are solid polymers that melt when heated. After applied to the substrate, the adhesive hardens by cooling. Examples of thermoplastic adhesives are hot-melt adhesives used in packaging. Elastomeric adhesives are made from polymeric resins having high degree of elongation and compression. They are hyper-elastic materials that return rapidly to their initial dimensions after the removal of the applied load. Hybrid adhesives are made by combining thermoplastic, thermosetting or elastomeric resins. This combination makes use of the best properties of each resin. In general, if high temperature rigid resins are combined with flexible tough elastomers, improved peel strength and energy absorption can be obtained. Recent development in hybrid adhesive systems resulted in improved peel strength and toughness of thermosetting resins without any reduction in their high temperature properties.

### 1.3.3. Classification by Method of Reaction

Adhesives are classified by method of reaction as chemical reaction, loss of solvent, loss of water and cooling from melting. They may solidify by losing solvent or hardening due to heat or chemical reaction. Adhesives that harden by chemical reaction may be further classified according to the type or the use: (1) two parts systems, (2) single part cured via catalyst or hardener, (3) moisture cured, (4) radiation cured such as light, ultraviolet and electron beam, (5) catalyzed by substrates and (6) in solid form such as tape, film and powder. Adhesives that harden by loss of solvent or water are classified as: (1) contact, (2) pressure sensitive, (3) reactivatable and (4) resinous. Adhesives that harden by cooling from melting are classified as: (1) hot melt and (2) hot melt pressure sensitive.

### 1.3.4. Classification by Physical Form

Adhesives are available in many physical forms and may be clas-

sified as liquid or paste multiple part solvent-less, liquid or paste one part solvent-less, liquid one part solution and solid including powder, tape, film, etc. Some types of adhesives are available in many forms, e.g. epoxy adhesive. A simpler classification of adhesives by physical form would be solid and liquid. Solid adhesives can be further classified as (1) film, (2) tape, (3) solid powders, (4) solved and primers and (5) hot melt. Liquid adhesives are either pure 100% solid paste and liquid or 100% solid paste and liquid with solvent to reduce viscosity. The first type can be further classified as (1) one component cured by heat, surface or anaerobic catalysts, or ultraviolet light or radiation and (2) two components cured at room temperature or heat. The second type is classified as (1) solvent-based contact, (2) water-based and (3) pressure sensitive.

### **1.3.5. Classification by Cost**

Cost plays an important role in the selection of an adhesive; therefore, it might be used as a method of classification even in an indirect way. The cost of using adhesives is not just the price of adhesive materials, but it should include other costs that are necessary to produce an adhesive joint. Thus, costs of labor, equipment, curing time, loss due to defective joints should be considered. In analyzing the real cost of an adhesive joint, the following parameters should be considered: (1) efficiency of bonding, (2) ease of application and requirement of equipment, such as jigs, ovens, pressers and fixtures, (3) total processing time for preparation of substrates, assembly, drying and curing, (4) cost of labour, (5) waste of adhesives and (6) rejected or defected joints.

### **1.3.6. Classification by End Use**

Adhesives are classified by end used, such as substrate type that will be jointed and environments for which they are suited. The substrate type can be, for instance, metal, composite, wood, etc. The environment classification include acid-resistant, heat-resistant and weather-able adhesives.

## **1.4. APPLICATIONS OF ADHESIVE BONDING**

Adhesive bonding has been successfully used in many industrial applications. In order to choose an adhesive system for a specific applica-

tion many factors should be considered. Strength, fatigue resistance, durability and expected lifetime are examples of such factors. The classifications presented in the previous section would be very helpful to evaluate the suitability of an adhesive system for a certain application. In industrial applications, structural reactive adhesives provide high strength, durability and temperature resistance. In the following sections, some important industrial applications are reviewed.

#### 1.4.1. Aerospace

In 1970, the U.S. Air Force funded the project Primarily Adhesive Bonded Structures Technology (PABST), in which aluminium aircraft structures were bonded with adhesives. The aim of this project was to produce repeatable and reliable bonding by determining the optimum joint design, the best surface treatment procedures and procedures for storage and application of adhesives. A summary of the type of adhesives used to bond different aircraft components is given in Table 1.4. Due to the importance of light weight in aircraft, adhesive systems usually are supplied as syntactics, which are types of foam materials. In general, the types of adhesives used for aircraft structures are one-component liquids and pasta cured by heat, two-component liquids and pasts, films and pressure sensitive. Epoxy pasta and film, which are cured in heated presses, are used to bond honeycomb panels and to bond skins to honeycomb. Honeycomb panels are fabricated from aluminium sheets in a sandwich structure bonded to a honeycomb core. In the construction of aileron at the trailing edge of the wing, laminate structures are made of epoxy graphite plies. Adhesives are used not only in bonding critical aircraft components such as fuselage and fuel tanks, but also to a great extent in bonding and sealing interior components, such as panels, seats, tray tables, overhead bins, galleys, toilets, etc.

**TABLE 1.4. The Use of Adhesive in Aircraft Structures [5].**

Bonded Structures	Adhesive Type
Metal honeycombs and skins	Epoxy
Metal honeycombs and skins for high temperature situations	Nitrile phenolic
Interior plastics	Acrylic, Pressure sensitive adhesives
Sealing interior plastics	Silicone
Plastics and composites	Polyurethane

**1.4.2. Automotive**

During the last few decades, the application of adhesive bonding to automotive industry has been significantly increased due to the new requirements of using thin panels of metals, such as steel, aluminium and magnesium, and the increased in the use of plastic. Adhesives are used in many of the essential components of a vehicle, such as the powertrain, body, trim, electrical system and brakes. A summary of the applications and adhesive types used for each component is given in Table 1.5. There are many applications for adhesives and sealants in

**TABLE 1.5. The Use of Adhesives in Automotive [5].**

<b>Component</b>	<b>Applications</b>	<b>Adhesive/Sealant Type</b>
Powertrain	Threadlocking	Anaerobics
	Oil pan gasketing, Rocker cover gasket	Silicones
	Oil filter assembly	Plastisols, cyanoacrylates
	Clutch facings	Phenolics
Body	Hem flange bonding	Plastisols, epoxies, polyurethane
	Body-in-white sealing	Plastisols, rubbers, Polyvinyl Chloride
	Roof bow joints	Rubber
	Anti-flutter stiffeners	SBR, polybutadiene
	Plastic panel bonds	Epoxies, polyurethane
	Bumper bonding	Reactive acrylics, polyurethane
Trim	Cavity sealants	Expandable polyurethane
	Labels, decals	Acrylic pressure sensitive
	Rear parcel shelves	Hot melts, reactive hot melt polyurethanes
	Sun visors, Floor insulation	Hot melts
	Mirrors	Silicones
	Upholstered seats, Headliner	Hot melts, reactive hot melt polyurethanes
	Dashboard	Polyurethane, hot melts
	Carpet bonding	Hot melts, pressure sensitive
Electrical systems	Windshield bonding	Polyurethane
	Headlight units	Epoxies, ultraviolet acrylics
	Rearlight units	Silicones, hot melts
	Spark plug seals	Silicones
Brakes	Motor bonding	Anaerobics, reactive acrylics, ultraviolet acrylics
	Disk pad bonding	Phenolics



# Finite Element Analysis

## 3.1. INTRODUCTION

**T**HIS chapter is intended to provide a basic background for the Finite Element Analysis (FEA). It concentrates on topics that are required for the analysis of adhesively bonded joints, which will be presented in later chapters. A brief history of FEA in general and of ANSYS package in particular is presented. As the analysis of bonded joints will be in two-dimensional space, the theoretical aspects of two-dimensional solid structural elements are briefly reviewed. Specific modelling techniques, which are directly related to FEA of adhesively bonded joints, such as geometric and material non-linearities, modelling of singularities and cracks, extracting fracture mechanics parameters from FEA results and modelling of Cohesive Zone Model, are summarized. Because heat transfer and diffusion problems will be presented in other chapters, the formulation of two-dimensional heat transfer conduction element is reviewed. FE formulation for steady state and transient heat transfer and diffusion analyses is briefly presented. Furthermore, ANSYS parametric design language, which is required for modelling and analysing adhesively bonded joints in later chapters, is reviewed.

## 3.2. A BRIEF HISTORY OF FEA

Although originally developed for stress analysis in complex air-frame structures, FEA currently is applicable to the broad field of continuum mechanics and many other disciplines. FEA is receiving much attention in academia and industries because of its diversity, flexibility

and ability to provide approximated numerical solutions to a wide range of engineering applications. The ideas of FEA are shared between three different specialists, namely an applied mathematician, a physicist and an engineer. At a certain point in the FEA history, each of these three specialists has independently developed the essential ideas of FEA for different reasons. The applied mathematicians were interested in solving boundary value problems of continuum mechanics by finding approximate upper and lower bounds for eigenvalues. The physicists were concerned with also solving continuum problems by obtaining piecewise approximate functions, whereas the engineers were interested in solving complex problems in aerospace structures and finding the stiffness of shell structures reinforced by ribs and spars.

In the applied mathematics literature in 1943, Courant [34] used for the first time piecewise continuous functions defined over triangular elements to study Saint-Venant torsion problem. In 1959, Greenstadt [35] presented an approach in which a continuous problem was reduced to a discrete problem. He used a discretization approach involving cells instead of points so the solution domain was divided into sets of subdomains. Since the late 1960s, the popularity of FEA has grown and applied mathematicians increased their studies in different aspects, including estimation of discretization error, rate of convergence and stability of different types of FEA formulation.

During almost the same period mentioned above, physicists also were busy with their FEA ideas. In 1947, Prager and Synge [36] published their development of the hyper-circle method, which originally was developed in conjunction with the classical theory of elasticity. Synge [37] demonstrated the hyper-circle method can be applied to continuum problems in a way similar to FEA.

The engineering community has for the first time adopted FEA ideas in the 1930s, when a structural engineer analyzed a truss problem by solving components of stress and deflection. This was done by considering the truss as an assembly of rods whose elastic behavior was well known. Then, by combining the individual elements, applying equilibrium equations and solving the resulting system of equations, unknown forces and deflection would be obtained for the whole structure. In order to apply this concept to an elastic continuum structure, Hrenikoff [38], in 1941, divided the continua into beam elements interconnected at a finite number of nodes. Using this concept, the problem can be solved in a way similar to that used for the truss. Further development of Hrenikoff's idea was reported by McHenry [39] in 1943, Kron [40,41] in

1944 and Newmark [42] in 1949. Starting from 1954 to around 1960, Argyris [43–46] presented techniques to deal with linear structural analysis and efficient solution for automatic digital computation. In 1956, Turner et al [47] have presented the first solution of plane stress continuum problems using triangular and elasticity theory. They have introduced for the first time the direct stiffness method, which was used to determine the finite element properties. The term “Finite Element Method” (FEM) was firstly used by Clough [48] in 1960 for solving plane elasticity problems. The term “Finite Element Analysis” (FEA) can be considered as the practical application of FEM. In 1965, Zienkiewicz and Cheung [49] reported that FEA was applicable to all fields that can be treated with variational form principles. Since the 1960s, the interest in FEA among engineers and scientists has grown quickly. A tremendous amount of technical papers has been published; a large number of conferences have been organized; and numerous books on the topic have appeared. A simple search in Google search engine on the phrase “Finite Element” results in approximately 13,200,000 results and a search in the web of science on the topic “Finite Element” results in approximately 423,231 articles.

### **3.3. A BRIEF HISTORY OF ANSYS PACKAGE**

In 1970, Swanson Analysis Systems Inc. (SASI) founded ANSYS. One year later, the first version of ANSYS software, ANSYS 2.0, was released. Geometric non-linearities and thermo-electric elements have been included in ANSYS since 1975. In 1981, SASI used the first workstation as an alternative to main frames. Few years later, in 1983, electro-magnetic analysis was introduced and the next version of ANSYS with electromagnetic capabilities was released. In 1985, SASI has started to offer online help and ANSYS software has incorporated parametric analysis and structural optimization. Color graphics and layered composite solid elements were introduced in 1987. In 1991, general purpose CFD solver for unstructured grids and TGrid tetrahedral mesher were released. In 1993, a new ANSYS software, namely Machine Design’s best Finite Element Analysis software for workstation, was released. In 1994, SASI renamed ANSYS, Inc. In 1996, ANSYS became a public company listed on the National Association of Securities Dealers Automated Quotations, NASDAQ (ANSS), and released DesignSpace with ANSYS workbench environment, ANSYS and LS-DYNA for crash and drop test simulations and commercial CFD with parallel processing.

Automatic contact detection for assemblies was introduced in 1998. A year later, ANSYS acquired Centric Engineering Systems Inc., adding multi-physics modelling and high-performance processing capabilities. Furthermore, in 2000, ANSYS acquired the meshing and post-processing tools of ICFE Engineering and, in 2001, the designXplorer tool for post-processing of CADOE S.A. Semi-automated easy-to-use moving deformed mesh, multi-domain material phase model for unstructured mesh and commercial k-turbulence model for unstructured mesh was added in 2001. In 2003 (ANSYS 7.1 and 8.0), ANSYS acquired CFX and released automated dynamic mesh capability and feature based mesh adaption for CFD. Solving hundred million degrees of freedom made possible in 2004. In 2005, the seamlessly coupled fluid-structure interaction (FSI) was introduced, and ANSYS acquired Century Dynamics Inc. and Harvard Thermal Inc. adding explicit dynamics and electronics cooling analysis tools. In 2006, ANSYS acquired Fluent Inc. and its fluid dynamics tools. Integrated rigid and flexible multi-body dynamics was launched in 2007. In 2008, ANSYS acquired Ansoft Corporation introducing high performance electronics software. In 2009 (ANSYS 12.0), ANSYS released the next-generation of ANSYS workbench and in 2010 ANSYS celebrated its 40th anniversary.

Nowadays, ANSYS becomes a comprehensive Finite Element software that contains more than 100,000 lines of codes. It has the capabilities to perform a wide range of analysis disciplines, such as static, dynamic, heat transfer electro-magnetic, etc. ANSYS is used in many engineering applications, including aerospace, mechanical, civil, marine, electronic and nuclear. ANSYS Graphical User Interface (GUI) has been evolved during the years and its current form consists of a graphic window, a main menu, an utility menu and a toolbar.

### **3.4. STRUCTURAL MECHANICS PROBLEMS**

#### **3.4.1. Two-dimensional Solid Element**

##### *3.4.1.1. Linear Elements*

In displacement formulation FEA, for two-dimensional solid element, the degrees of freedom are the displacements in two directions, e.g. in the  $x$  and  $y$  directions if Cartesian coordinates are considered. The first step in formulating two-dimensional element is to express the functions of degrees of freedom in terms of their nodal values. In other

words, the displacement functions are interpolated at any position in the element using the nodal degrees of freedom. To do so, interpolation functions, known as shape functions, should be defined for each type of element. To illustrate this step, consider the straight-sided triangular element, shown in Figure 3.1, with three nodes at each corner, labelled as  $I, J$  and  $K$  proceeding anti-clockwise from an arbitrary chosen node  $I$ . Each node has two degrees of freedom, namely  $u_x$  and  $u_y$ , and its position is defined by two global coordinates, namely  $x$  and  $y$ . Thus, nodes  $I, J$  and  $K$  have  $(u_{xi}, u_{yi}), (u_{xj}, u_{yj})$  and  $(u_{xk}, u_{yk})$ , respectively, and are located at  $(x_i, y_i), (x_j, y_j)$  and  $(x_k, y_k)$ , respectively. The variation of the degrees of freedom  $u_x$  and  $u_y$  inside the element is assumed to be linear and is given by:

$$u_x = \alpha_1 + \alpha_2 x + \alpha_3 y \tag{3.1}$$

$$u_y = \alpha_4 + \alpha_5 x + \alpha_6 y \tag{3.2}$$

Where the constants  $\alpha_1$  to  $\alpha_6$  can be obtained from the nodal displacements and coordinates conditions.

For the element shown in Figure 3.1, there are six nodal displacements and coordinates conditions, which are given by:

$$\begin{aligned} u_x = u_{xi} \text{ at } (x_i, y_i) & \quad u_x = u_{xj} \text{ at } (x_j, y_j) & \quad u_x = u_{xk} \text{ at } (x_k, y_k) \\ u_y = u_{yi} \text{ at } (x_i, y_i) & \quad u_y = u_{yj} \text{ at } (x_j, y_j) & \quad u_y = u_{yk} \text{ at } (x_k, y_k) \end{aligned} \tag{3.3}$$

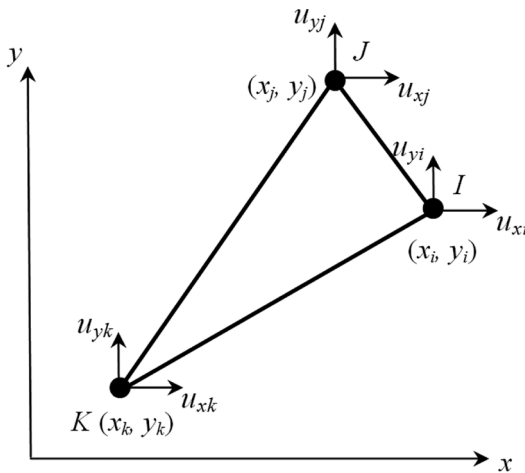


FIGURE 3.1. Triangular element in Cartesian coordinates.

# Stress Analysis

## 5.1. INTRODUCTION

**A**FTER constructing the FE models of the bonded joints in Chapter 4, stress analysis for these joints is carried out in this chapter. In all analyses considered in this chapter, non-linear elastic plastic stress-strain behavior is defined for the adhesive material. The substrates are chosen to be either metallic (Aluminium or Steel) or CRFP composite unidirectional or multidirectional laminates. As most joints show high rotation due to the offset of loading, large deformation should be activated in the analysis. Therefore, both material nonlinearity (see section 3.4.2.2 in Chapter 3) and geometric nonlinearity (see section 3.4.2.1) should be taken into account during the solution of the FEA equations. The load is applied in different sub-steps and the solution is obtained incrementally as explained in section 3.4.2.3. Four joints are analyzed in this chapter, namely SLJ, DLJ, LSJ and BJ, which are the four un-cracked joints considered in Chapter 4. For stress analysis of each joint, two files are created and supplied with this book. The first file has the name format ‘jointname\_Smodel.inp’ and contains the definition of material properties, geometric parameters, keypoints, meshing, etc., as explained in Chapter 4. The second file, which calls the first file with /INPUT command, applies loads and boundary conditions, and performs the stress analysis. This stress analysis file has the name format ‘jointname\_stress\_analysis.inp’. It should be noted that both files should be in the same directory so that ANSYS can find the first file when it is imported in the second file.

## 5.2. STRESS ANALYSIS USING ANSYS

### 5.2.1. Single Lap Joint—Metallic substrates

For SLJ, the file 'SLJ\_Smodel.inp' is imported to the stress analysis file, 'SLJ\_stress\_analysis.inp' before applying boundary conditions and loads, and solving the problem. Then, we define one additional parameter, namely the applied force in Newton, which is acting at both ends of the substrates and distributed over the thickness and the width of the substrates as shown in Figure 5.1. ANSYS commands for these two steps are:

/INPUT,SLJ_Smodel,inp	! Import SLJ model
ft=200	! Define total applied load in N

Further a title is defined, using /TITLE command, as:

/TITLE,Single Lap Joint - Stress Analysis	! Define a title for the analysis
---	-----------------------------------

All commands related to applying boundary conditions and load should be issued in the solution processor, which can be entered through the command /SOLU. The SLJ is modelled assuming simply supported boundary conditions by constraining one keypoint (and node) at the left hand side of the lower substrate in the x and y directions (all DOF) and one keypoint at the right hand side of the upper substrate in the y directions (only uy). This is done in ANSYS by selecting the relevant keypoint using KSEL command without the need to know the number of each keypoint. ANSYS commands for applying boundary conditions are:

/SOLU	! Enter solution processor
KSEL,s,loc,x,0	! Select keypoints at x=0, left edge of the lower substrate
KSEL,r,loc,y,0	! Select from previous selection keypoints at y=0 (only one keypoint is selected)
DK,all,all	! Constrain all DOF at selected keypoint
KSEL,s,loc,x,2*lt-lo	! Select keypoints at x=2lt-lo, right edge of the upper substrate
KSEL,r,loc,y,tadh+tadv	! Select from previous selection keypoints at y=tadh+tadv (only one keypoint is selected)
DK,all,uy	! Constrain displacement in the y direction at selected keypoint



FIGURE 5.1. SLJ mesh, applied pressure and boundary conditions.

Alternatively, keypoints may be explicitly defined by their numbers. In order to include the geometric nonlinearity, the command NLGEOM is issued to switch on the effect of large deflection. The command LNSRCH also is issued to activate a line search to be used with Newton-Raphson iteration method. This is useful for the convergence of the nonlinear solution, especially for bonded joints where both geometric and material nonlinearities are present. ANSYS commands for these two steps are:

NLGEOM,on	! Include geometric nonlinearity (large deformation) in the analysis
LNSRCH,on	! Activate a line search for Newton-Raphson

For non-linear analysis, the load is applied in steps so that the solution is performed in sub-steps. If only one load step is applied as in our case, the load step starts at time 0 and ends at time 1. The load varies linearly as a function of time as shown in Figure 5.2. The solution then will be obtained at different time sub-steps, i.e. different load sub-steps. To specify the time at the end of a load step in ANSYS, the command TIME is used. The linear variation of loading (ramped) is applied using the command KBC. Although the default of the command is ramped loading, it is given herein for completeness:

TIME,1	! Set time for load step to 1
KBC,0	! Apply ramped loading within this load step

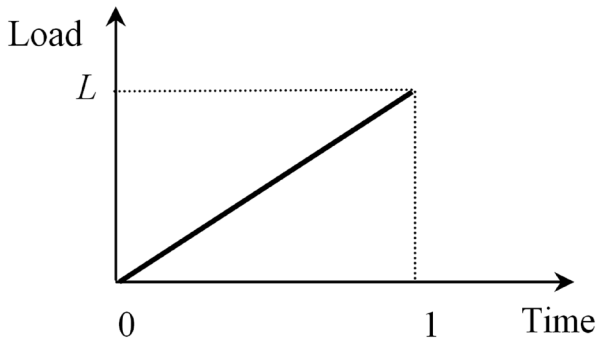


FIGURE 5.2. Load as a function of time for nonlinear analysis.



The load is applied to the left end of the lower substrate and to the right end of the upper substrate as a pressure on lines, which are selected through their connectivity's to keypoints. First, the keypoints at the right edge of the upper substrate and at the left edge of the lower substrate are selected using KSEL commands, and then the lines attached to these keypoints are selected using LSLK commands. The pressure is applied on the selected lines using SFL commands.

KSEL,s,loc,x,2*Lt-Lo	! Select keypoints at $x=2lt-lo$ , right edge of the upper substrate
KSEL,a,loc,x,0	! Additional selection of keypoints at $x=0$ , left edge of the bottom substrate
LSLK,s,1	! Select lines containing the selected keypoints
SFL,all,pres,-ft/(tadh*wt)	! Apply pressure at the selected line ( $p=-ft/(tadh*wt)$ )
KSEL,all	! Select all keypoints
LSEL,all	! Select all lines

For solving the non-linear problem, the user may either specify specific time sub-steps or ask ANSYS to automatically calculating time for each sub-step based on the convergence of the previous sub-step. For the latter option, automatic time stepping should be activated using the command AUTOTS. In such a case, starting, minimum and maximum time steps should be defined using the command DELTIM.

AUTOTS,on	! Activate automatic time stepping
DELTIM,0.2,0.02,0.2	! Specify time step sizes with 0.2, 0.02, and 0.2 as starting time step, minimum time step and maximum time step, respectively

Writing all solution items and printing them for every sub-step are done using the commands OUTRES and OUTPR, respectively:

OUTRES,all,all	! Write all solution items
OUTPR,all,all	! Print solution for all items for every sub-step

The above steps are necessary if the solution at every sub-step is required. Finally, this load step is solved, the solution solver is exited and the database is saved using the following ANSYS commands:

SOLVE	! Solve load step
FINI	! Exit solver
SAVE	! Save database

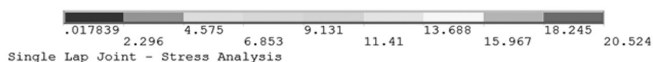
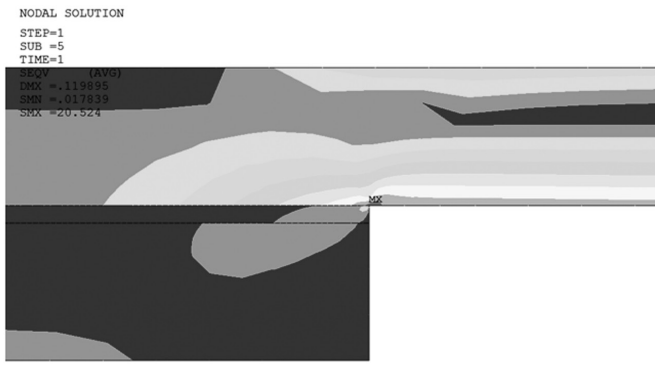


FIGURE 5.3. Deformed shape for SLJ.

The results can be easily post processed using the interactive menu, but they also can be programmed in APDL. As an example for plotting the deformed shape of the SLJ shown in Figure 5.3, for which the following commands are used:

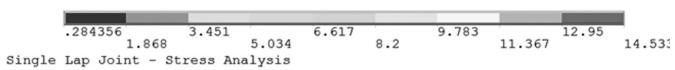
/POST1	! Enter post-processor
/DSCALE,1,AUTO	! Scale displacements automatically
PLDISP,1	! Display the deformed structure

Plots of von-Mises stress contour for SLJ are shown in Figure 5.4.



Single Lap Joint - Stress Analysis

(a)



Single Lap Joint - Stress Analysis

(b)

FIGURE 5.4. von-Mises contour plots for SLJ (a) adhesive and substrates and (b) only adhesive.

### 5.2.2. Double Lap Joint—CFRP Composite Substrates

The input file for DLJ stress analysis, 'DLJ\_stress\_analysis.inp', starts with calling the file containing the DLJ model, 'DLJ\_Smodel.inp', developed in Chapter 4, using /INPUT command:

/INPUT,DLJ_Smodel,inp	! Import DLJ model
-----------------------	--------------------

The applied load in Newton's is defined as a parameter:

ft=1200	! Define total applied load in N
---------	----------------------------------

After entering the solution processor, boundary conditions can be applied. A summary of boundary conditions and applied load is illustrated in Figure 5.5. The middle substrate of the DLJ is fixed at the left hand side:

KSEL,s,loc,x,0	! Select keypoints at x=0, left edge of the middle substrate
DK,all,all,0,,1	! Constrain all DOF at all selected keypoints

Due to symmetry the mid-plane of the middle substrate is constrained in the y-direction:

KSEL,s,loc,y,0	! Select keypoints at y=0, symmetric line
DK,all,uy,0,,1	! Constrain uy at all selected keypoints, symmetric boundary conditions

One node is constrained in the y-direction at the right edge of the upper substrate by constraining one keypoint using KSEL command:

KSEL,s,loc,x,l1+l2-lo	! Select keypoints at right edge of the upper substrate
KSEL,r,loc,y,tadh/2+tadv	! Select from previous selection keypoints at y= tadh+tadv (only one keypoint is selected)
DK,all,uy	! Constrain displacement in the y direction at selected keypoint



FIGURE 5.5. DLJ mesh, applied pressure and boundary conditions.

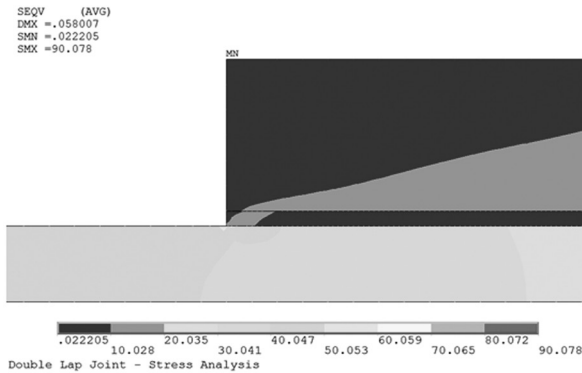


FIGURE 5.6. Deformed shape for DLJ.

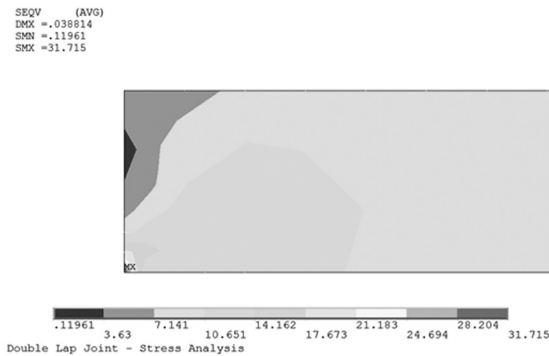
Load is applied as a pressure on a line at right edge of the upper substrate using SFL command:

KSEL,s,loc,x,11+12-lo	! Select keypoints at right edge of the upper substrate
LSLK,s,1	! Select lines containing the selected keypoints
SFL,all,pres,-ft/(tadh*wt)	! Apply pressure at the selected line ( $p=-ft/(tadh*wt)$ )

Plots of deformed shape and von-Mises stress contour for DLJ are shown in Figures 5.6 and 5.7, respectively.



(a)



(b)

FIGURE 5.7. von-Mises contour plots for DLJ (a) adhesive and substrates and (b) only adhesive.



FIGURE 5.8. LSJ mesh, applied pressure and boundary conditions.

### 5.2.3 Lap Strap Joint—CFRP Composite Substrates

Similar to the previous two joints, the stress analysis of LSJ is carried out using the input file ‘LSJ\_stress\_analysis.inp’, which calls the LSJ model developed in Chapter 4:

```
/INPUT,LSJ_Smodel,inp      ! Import LSJ model
```

Applied load is defined as:

```
ft=1500                    ! Define total applied load in N
```

Boundary conditions and applied load for LSJ are summarised in Figure 5.8. The joint is completely fixed at the right edge of the upper substrate:

```
KSEL,s,loc,x,11+12      ! Select keypoints at x=11+12, right edge of the
                        ! upper substrate
DK,all,all,0,,1        ! Constrain all DOF at all selected keypoints
```

And it is only constrained in the y-direction at the left edge of the lower substrate:

```
KSEL,s,loc,x,0         ! Select keypoints at x=0, left edge of the lower
                        ! substrate
DK,all,uy,0,,1        ! Constrain uy at all selected keypoints
```

Load is applied as a pressure on a line at the left edge of the lower substrate using SFL command:

```
KSEL,s,loc,x,0         ! Select keypoints at x=0, left edge of the lower
                        ! substrate
LSLK,s,1               ! Select lines containing the selected keypoints
SFL,all,pres,-ft/(tadh*wt) ! Apply pressure at the selected line (p=-ft/
                        ! (tadh*wt))
```



FIGURE 5.9. Deformed shape for LSJ.

# Appendix A

## Summary of ANSYS Files\*

### Single Lap Joint

#### *Stress Analysis*

SLJ\_Smodel.inp

SLJ\_stress\_analysis.inp

#### *Fracture Mechanics Analysis—Interface Crack*

SLJ\_interface\_crack.inp

SLJ\_fracture\_analysis\_ic.inp

SERR\_SLJ\_ic.inp

#### *CZM Analysis—Contact Elements*

SLJ\_CZM\_model\_cont.inp

SLJ\_CZM\_analysis\_cont.inp

#### *CZM Analysis—Interface Elements*

SLJ\_CZM\_model\_inter.inp

SLJ\_CZM\_analysis\_inter.inp

#### *Fatigue Crack Propagation*

SLJ\_fatigue\_CP.inp

SERR\_SLJ\_fat.inp

#### *Thermal Analysis*

SLJ\_Tmodel.inp

SLJ\_thermal\_analysis.inp

\*Files listed in this Appendix can be downloaded for use with ANSYS. Instructions for executing the download can be found on page i at the front of this book.

*Coupled Thermal-Stress Analysis*

SLJ\_Tmodel.inp  
SLJ\_thermal\_analysis.inp  
SLJ\_Smodel.inp  
SLJ\_coupled\_analysis.inp  
Interp\_mat\_SLJ.inp  
Input\_mat\_SLJ.inp

**Double Lap Joint**

*Stress Analysis*

DLJ\_Smodel.inp  
DLJ\_stress\_analysis.inp

*Fracture Mechanics Analysis—Adhesive Crack*

DLJ\_adhesive\_crack.inp  
DLJ\_fracture\_analysis\_ac.inp  
SERR\_DLJ\_ac.inp

**Lap Strap Joint/Cracked Lap Shear**

*Stress Analysis*

LSJ\_Smodel.inp  
LSJ\_stress\_analysis.inp

*Fracture Mechanics Analysis—Interface Crack*

CLS\_interface\_crack.inp  
CLS\_fracture\_analysis\_ic.inp  
SERR\_CLS\_ic.inp

*CZM Analysis—Contact Elements*

LSJ\_CZM\_model\_cont.inp  
LSJ\_CZM\_analysis\_cont.inp

*Diffusion Analysis*

LSJ\_Dmodel.inp  
LSJ\_diffusion\_analysis.inp

*Coupled Diffusion-Stress Analysis*

LSJ\_Dmodel.inp  
LSJ\_diffusion\_analysis.inp

LSJ\_Smodel.inp  
LSJ\_coupled\_analysis.inp  
Interp\_mat\_LSJ.inp  
Input\_mat\_LSJ.inp

### **Butt Joint**

#### *Stress Analysis*

BJ\_Smodel.inp  
BJ\_stress\_analysis.inp

#### *Diffusion Analysis*

BJ\_Dmodel.inp  
BJ\_diffusion\_analysis.inp

#### *Coupled Diffusion-Stress Analysis*

BJ\_Dmodel.inp  
BJ\_diffusion\_analysis.inp  
BJ\_Smodel.inp  
BJ\_coupled\_analysis.inp  
Interp\_mat\_BJ.inp  
Input\_mat\_BJ.inp

### **Double Cantilever Beam**

#### *Fracture Mechanics Analysis—Adhesive Crack*

DCB\_adhesive\_crack.inp  
DCB\_fracture\_analysis\_ac.inp  
SERR\_DCB\_ac.inp

#### *CZM Analysis—Contact Elements*

DCB\_CZM\_model\_cont.inp  
DCB\_CZM\_analysis\_cont.inp

#### *CZM Analysis—Interface Elements*

DCB\_CZM\_model\_inter.inp  
DCB\_CZM\_analysis\_inter.inp

#### *Fatigue Crack Propagation*

DCB\_fatigue\_CP.inp  
SERR\_DCB\_fat.inp





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